Initial Analysis in JMulTi

June 12, 2006

Helmut Lütkepohl, Markus Krätzig

The initial analysis provides a starting point for the time series analysis with JMulTi . It contains plots of important characteristics as summarized in the spectrum and autocorrelation functions and there are also tests for the order of integration as well as cointegration tests, which should help choosing an appropriate statistical model. More general tasks, like importing, manipulating and transforming time series, managing data sets, etc. are described in the help section JMulTi -General Help.

1   Plot Time Series
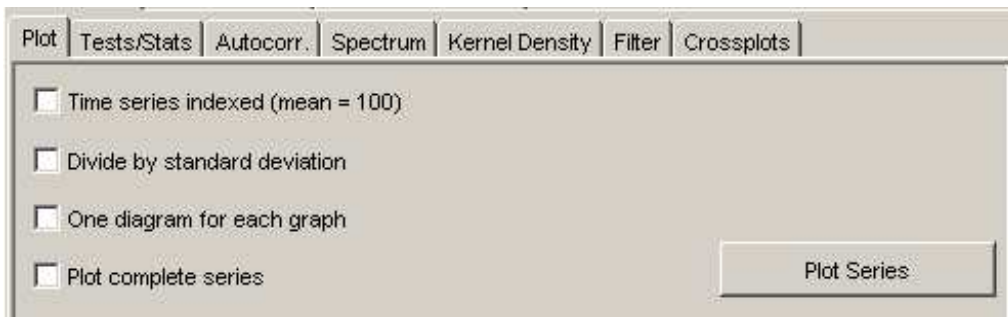
1.1   Specification in JMulTi

Figure 1: Specification of Time Series Plots

This panel can be used to configure plots of the selected time series.

time series indexed   The mean of the selected series is set to 100 and the observations are indexed relative to it. This feature is helpful to plot series with different magnitudes in the same diagram, e.g. interest rates and GNP.

divide by standard deviation   The selected series is standardized.

one diagram for each graph   Each graph is shown in its own diagram, otherwise one diagram is used for all graphs.

plot complete series   All available observations of all selected series are plotted together. If a series is shorter or has missing values then there will be holes in the respective graph. If not selected then only the selected sample period is plotted.

2 Descriptive Statistics
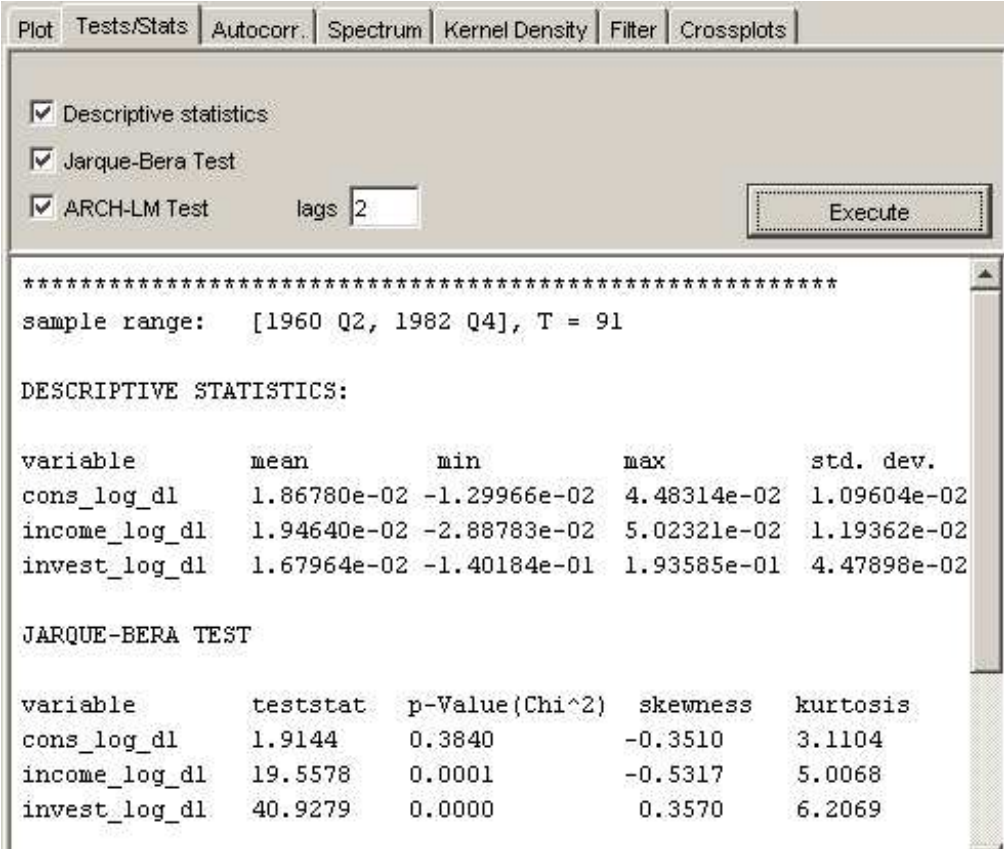
2.1 Specification in JMulTi

Figure 2: Specification of Descriptive Statistics

One may select one or more time series to get tables with the following statistics:

descriptive statistics   mean, minimum, maximum, standard deviation, variance

Jarque-Bera Test   for a description, see Sec. 13.2

ARCH-LM Test   for a description, see Sec. 13.2

3 Autocorrelation

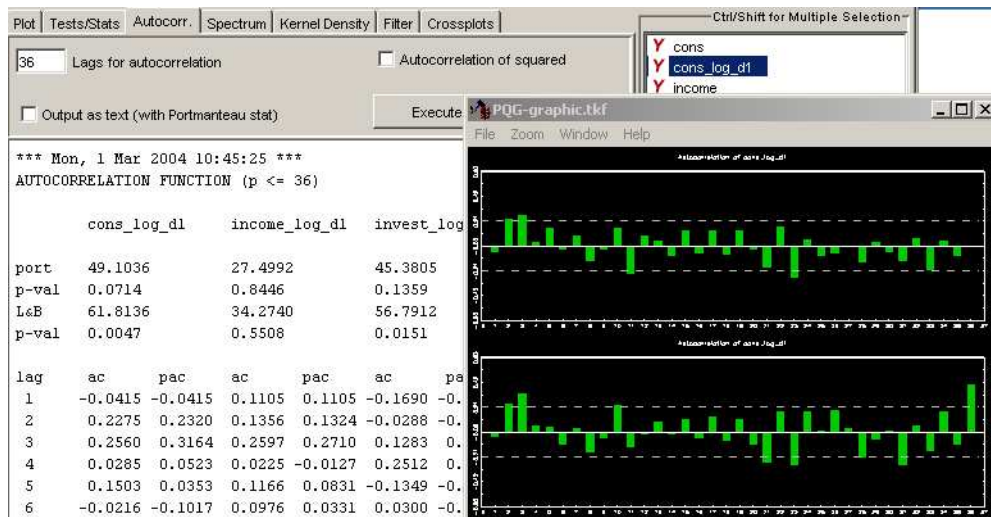## 3.1 Specification in JMulTi

Figure 3: Specification of AC/PAC

The autocorrelation and partial autocorrelation functions of the selected time series are given up to the maximum lag order specified.

AC/PAC for a description, see Sec. 13.2

Portmanteau Test for a description, see Sec. 13.2

4 Spectrum

4.1 Background

The ACs of a stationary stochastic process may be summarized compactly in the *spectral density function*. It is defined as

$$f_y(\lambda) = (2\pi)^{-1} \sum_{j=-\infty}^{\infty} \gamma_j e^{-i\lambda j} = (2\pi)^{-1} \left(\gamma_0 + 2 \sum_{j=1}^{\infty} \gamma_j \cos(\lambda j) \right) \tag{1}$$

where $i = \sqrt{-1}$ is the imaginary unit, $\lambda \in [-\pi, \pi]$ is the frequency and the γ_j are the autocovariances of y_t. It is estimated as

$$\hat{f}_y(\lambda) = (2\pi)^{-1} \left(\omega_0 \hat{\gamma}_0 + 2 \sum_{j=1}^{M_T} \omega_j \hat{\gamma}_j \cos(\lambda j) \right),$$

where the weights ω_j $(j = 1, \ldots, M_T)$ represent the so-called *spectral window* and M_T is the *truncation point*. In JMulTi the Bartlett window is used:

$$\omega_j = 1 - j/M_T \quad \text{(Bartlett (1950))}.$$

Choosing all $\omega_j = 1$ and $M_T = T - 1$ results in the periodogram. In JMulTi it is obtained by setting the window size to 1.

4.2 Specification in JMulTi

4.2.1 Input

Bartlett window size   sets the parameter M_T in 4.1, $M_T = 1$ produces the periodogram of the series.

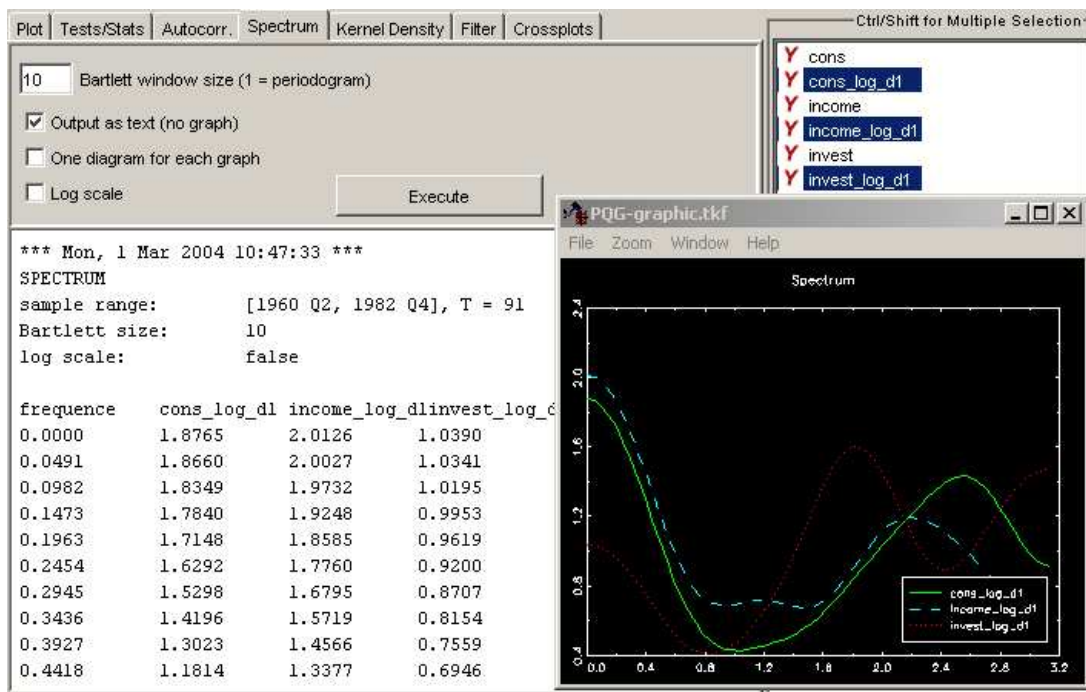**log scale**   use $\log \hat{f}_y(\lambda)$

Figure 4: Specification of Spectrum

5 Crossplots

5.1 Background

Sometimes it is useful to investigate the direct relationship between two variables. Crossplots offer an intuitive graphical tool to look at comovements between two different variables. It may also be helpful to compare the plot with a simple OLS regression line, as well as with a nonparametric estimate.

5.1.1 OLS Regression

Here a simple OLS regression is carried out, assuming that the model has the form $y_t = \alpha + \beta x_t + u_t$, where u_t is the regression error.

5.1.2 Nadaraya-Watson Regression

A possibly nonlinear regression function is assumed, $m(x) = \mathsf{E}(Y|X = x), x \in \Re$ with X being the design variable and Y the response variable. The Nadaraya-Watson estimator is defined as

$$\hat{m}(x) = \frac{\sum_{t=1}^{T} y_t K\left(\frac{x-x_t}{h}\right)}{\sum_{t=1}^{T} K\left(\frac{x-x_t}{h}\right)}.$$

K is a kernel function. The kernels available in JMulTi are specified in Sec. 6. h is the bandwidth. Here it is chosen automatically by

$$h = 0.9 T^{-1/5} \min(\hat{\sigma}_x, IQR/1.34), \quad \text{see Silverman (1986), Eq. (3.31),}$$

where $\hat{\sigma}_x$ is the standard deviation and IQR denotes the interquartile range of the x_t observations. As usual, T is the sample size.

The nonparametric estimation does not assume a special functional form for the model and can therefore capture possible nonlinearities in the relationship between X and Y.

5.2 Specification in JMulTi

5.2.1 Input

variables First select the variables that should be plotted against each other in the time series list. They will appear in the two tables for the x- and y-axis. Then one can click on a variable for each axis and invoke the plot.
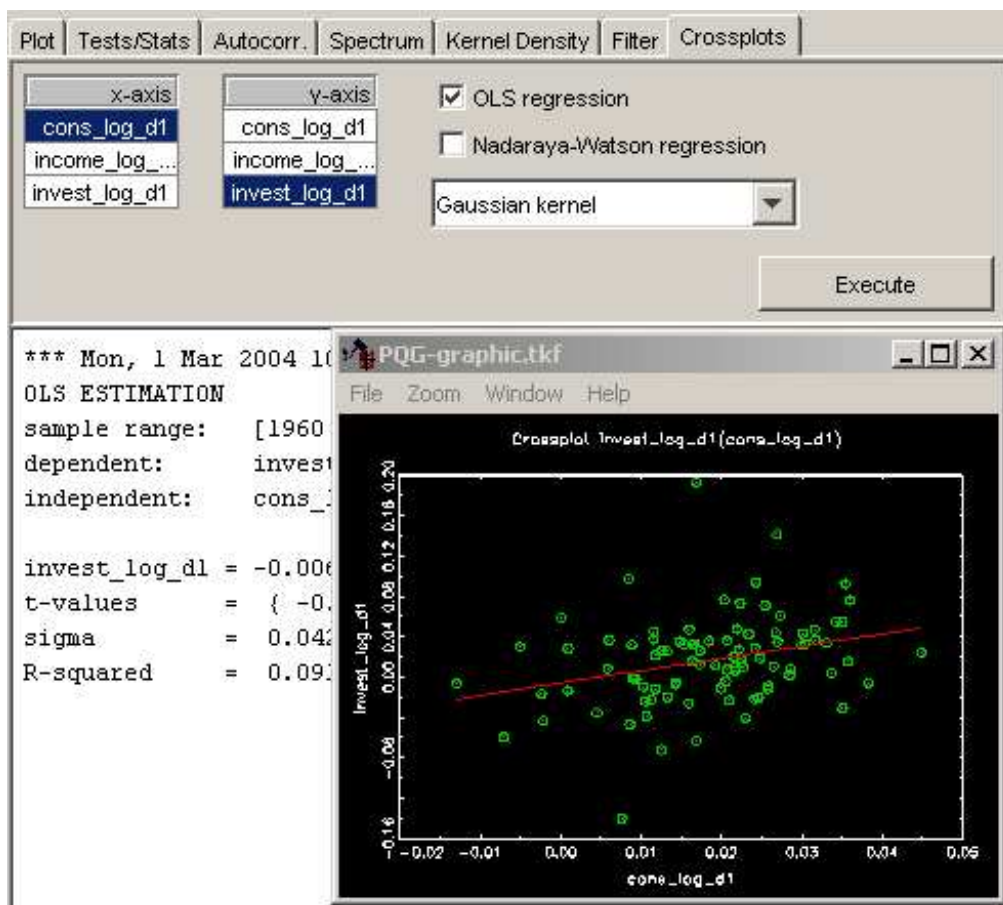
Figure 5: Specification of Crossplots

6 Kernels

Table 1: Kernels Available in JMulTi

Kernel	$K(u)$
Gaussian	$(2\pi)^{-1/2}\exp(-u^2/2)$
Biweight	$\begin{cases} \frac{15}{16}(1-u^2)^2 & \text{for } \lvert u\rvert < 1 \\ 0 & \text{otherwise} \end{cases}$
Rectangular	$\begin{cases} \frac{1}{2} & \text{for } \lvert u\rvert < 1 \\ 0 & \text{otherwise} \end{cases}$
Triangular	$\begin{cases} 1-\lvert u\rvert & \text{for } \lvert u\rvert < 1 \\ 0 & \text{otherwise} \end{cases}$
Epanechnikov	$\begin{cases} \frac{3}{4}(1-\frac{1}{5}u^2)/\sqrt{5} & \text{for } \lvert u\rvert < \sqrt{5} \\ 0 & \text{otherwise} \end{cases}$

7 Unit Root Tests

Because the order of integration of a time series is of great importance for the analysis, a number of statistical tests have been developed for investigating it. In JMulTi there are several tests implemented testing the null hypothesis that there is a unit root against the alternative of stationarity of a DGP which may have a nonzero mean term, a deterministic linear trend and perhaps seasonal dummy variables. The stochastic part is modeled by an AR process or, alternatively, it is accounted for by nonparametric techniques. Another test allows for the possibility of modeling structural shifts as they are observed. The KPSS test checks the null hypothesis of stationarity against an alternative of a unit root.

8 Augmented Dickey-Fuller (ADF) Test

8.1 Background

ADF tests are based on models of the form

$$\Delta y_t = \phi y_{t-1} + \sum_{j=1}^{p-1} \alpha_j^* \Delta y_{t-j} + u_t. \tag{2}$$

In this model the pair of hypotheses

$$H_0 : \phi = 0 \quad \text{versus} \quad H_1 : \phi < 0$$

is tested based on the t-statistic of the coefficient ϕ from an OLS estimation of (2) (Fuller (1976), Dickey and Fuller (1979)). H_0 is rejected if the t-statistic is smaller than the relevant critical value. If $\phi = 0$ (that is, under H_0) the series y_t has a unit root and is nonstationary, whereas it is regarded as stationary if the null hypothesis is rejected.

The test statistic has a nonstandard limiting distribution. Critical values have been obtained by simulation and they are available, for instance, in Fuller (1976) and Davidson and MacKinnon (1993). The limiting distribution depends on the deterministic terms which have to be included. Therefore, different critical values are used when a constant or linear trend term is added in (2). Also seasonal dummies may be included.

In these tests a decision on the AR order or, equivalently, on the number of lagged differences of y_t has to be made. This choice may be based on the model selection criteria or a sequential testing procedure may be used which eliminates insignificant coefficients sequentially starting from some high order model.

8.2 Specification in JMulTi

Input

selection of variables   one endogenous variable may be selected, exogenous or user defined deterministic variables are ignored

nonzero mean   constant added to regression

time trend   constant and trend term added to regression

seasonal dummies   seasonal dummies and constant added to regression

actual number of lags   the number of lagged differences $\Delta y_{t-1}, ..., \Delta y_{t-p}$ in the regression
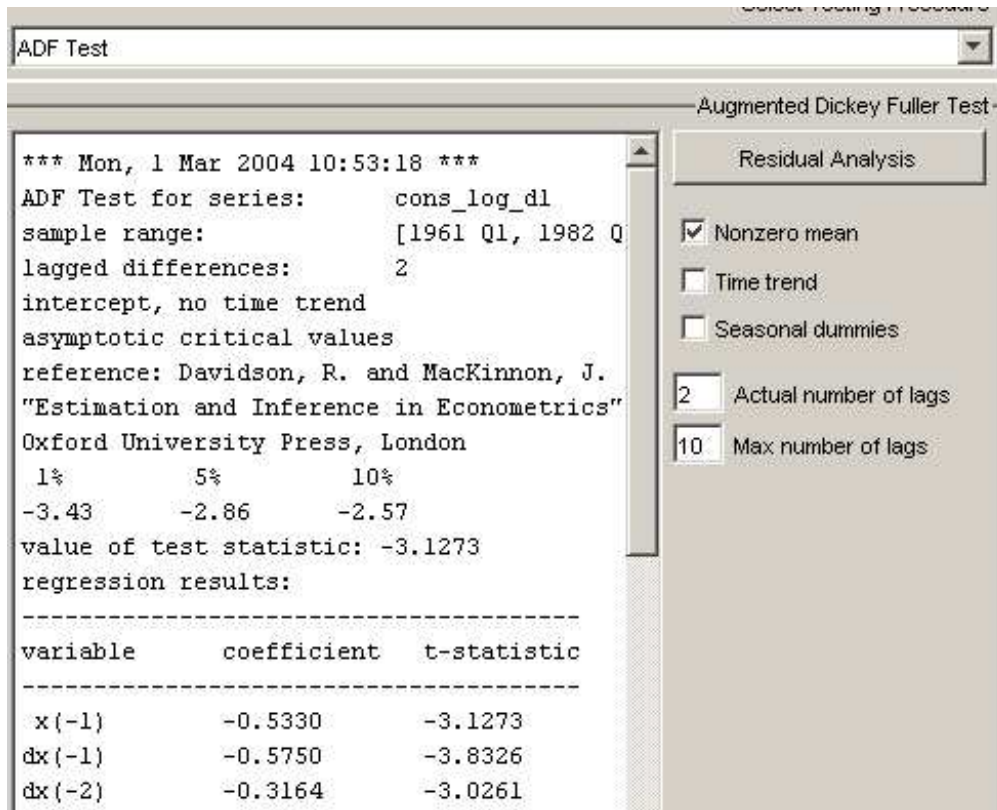
Figure 6: Specification of ADF Test

max number of lags   the number of lagged differences taken into account for the computation of the information criteria, max lag does not affect the sample period for the test regression, see also Sec. 14

residual analysis   see Sec. 13

Output

value of test statistic   the t-statistic of ϕ

dx(-i)   stands for Δy_{t-i}

sdummy(i)   the ith seasonal dummy

RSS   residual sum of squares $\sum_{t=1}^{T} \hat{u}_t^2$

information criteria   the optimal number of lagged differences for the respective criteria are presented, see Sec. 14

residual analysis   see Sec. 13

9 Schmidt-Phillips Test

9.1 Background

Schmidt and Phillips (1992) have proposed another variant of tests for the null hypothesis of a unit root when a deterministic linear trend is present. They suggest to estimate the deterministic term in a first step under the unit root hypothesis. Then the series is adjusted for the deterministic terms and a unit root test is applied to the adjusted series. Assuming that a deterministic linear trend term is present, the adjusted series is $\tilde{x}_t = y_t - \tilde{\mu}_0 - \tilde{\mu}_1 t$. In order to allow for more general DGPs than finite order AR processes, Schmidt and Phillips (1992) propose to base the test on a regression

$$\Delta \tilde{x}_t = \phi \tilde{x}_t + e_t,$$

where an adjustment factor is used in setting up the test statistic to account for the dependence structure of the error term e_t. For the present case, Schmidt and Phillips (1992) suggest the following two test statistics:

$$Z(\rho) = T \tilde{\phi} \tilde{\sigma}_\infty^2 / \tilde{\sigma}_e^2 \quad \text{and} \quad Z(\tau) = \frac{\tilde{\sigma}_\infty}{\tilde{\sigma}_e} t_{\tilde{\alpha}}.$$

Here $\tilde{\sigma}_e^2 = T^{-1} \sum_{t=1}^{T} \tilde{e}_t^2$ is the variance estimator based on the OLS residuals of the model $y_t = \nu_0 + \nu_1 t + \rho y_{t-1} + e_t$ and

$$\tilde{\sigma}_\infty^2 = \frac{1}{T} \sum_{t=1}^{T} \tilde{e}_t^2 + 2 \sum_{j=1}^{l_q} \omega_j \left(\frac{1}{T} \sum_{t=j+1}^{T} \tilde{e}_t \tilde{e}_{t-j} \right)$$

is a nonparametric estimator of the so-called long-run variance of e_t with ω_j being a Bartlett window, $\omega_j = 1 - \frac{j}{l_q+1}$. The asymptotic null distributions of these test statistics are different from those of the ADF statistics. Critical values for these tests are tabulated in Schmidt and Phillips (1992).

9.2 Specification in JMulTi

selection of variables   one endogenous variable may be selected, exogenous or user defined deterministic variables are ignored

statistics   One may choose between the two available statistics and adjust the number of lags, l_q, for the computation of the so-called long-run variance. One possible suggestion is to use $l_q = q(T/100)^{1/4}$ with $q = 4$ or $q = 12$.
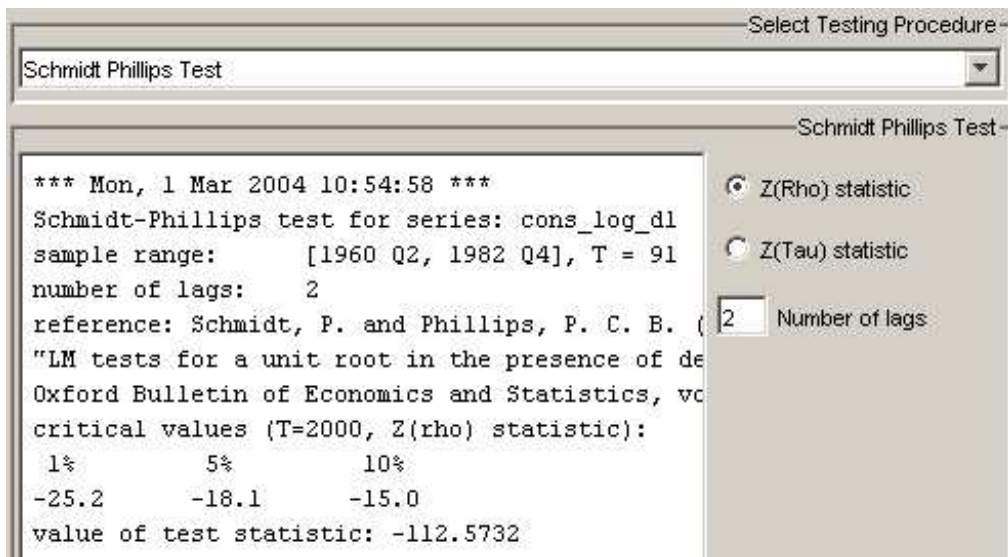
Figure 7: Specification of the Schmidt-Phillips Test

10 KPSS Test

10.1 Background

The integration properties of a series y_t may also be investigated by testing

$$H_0 : y_t \sim I(0) \quad \text{against} \quad H_1 : y_t \sim I(1),$$

that is, the null hypothesis that the DGP is stationary is tested against a unit root. Kwiatkowski, Phillips, Schmidt and Shin (1992) have derived a test for this pair of hypotheses. If there is no linear trend term, they start from a DGP

$$y_t = x_t + z_t,$$

where x_t is a random walk, $x_t = x_{t-1} + v_t$, $v_t \sim iid(0, \sigma_v^2)$, and z_t is a stationary process. In this framework the foregoing pair of hypotheses is equivalent to the pair $H_0 : \sigma_v^2 = 0$ versus $H_1 : \sigma_v^2 > 0$. Kwiatkowski et al. (1992) propose the following test statistic

$$KPSS = \frac{1}{T^2} \sum_{t=1}^{T} S_t^2 / \hat{\sigma}_\infty^2$$

where $S_t = \sum_{j=1}^{t} \hat{w}_j$ with $\hat{w}_t = y_t - \bar{y}$ and $\hat{\sigma}_\infty^2$ is an estimator of

$$\sigma_\infty^2 = \lim_{T \to \infty} T^{-1} \mathsf{Var} \left(\sum_{t=1}^{T} z_t \right),$$

that is, $\hat{\sigma}_\infty^2$ is an estimator of the long-run variance of the process z_t. Kwiatkowski et al. (1992) propose a nonparametric estimator for this quantity based on a Bartlett window with a lag truncation parameter $l_q = q(T/100)^{1/4}$:

$$\hat{\sigma}_\infty^2 = \frac{1}{T} \sum_{t=1}^{T} \hat{w}_t^2 + 2 \sum_{j=1}^{l_q} \omega_j \left(\frac{1}{T} \sum_{t=j+1}^{T} \hat{w}_t \hat{w}_{t-j} \right),$$

where $\omega_j = 1 - \frac{j}{l_q+1}$. Critical values may be found, e.g., in Kwiatkowski et al. (1992). The null hypothesis of stationarity is rejected for large values of $KPSS$.

If a deterministic trend is suspected, the point of departure is a DGP

$$y_t = \mu_1 t + x_t + z_t,$$

and the $\hat{w}_t$ are residuals from a regression

$$y_t = \mu_0 + \mu_1 t + w_t.$$

With these quantities the test statistic is computed in the same way as before. Its limiting distribution under H_0 is different from the case without trend term, however. Critical values for the case with trend are available from Kwiatkowski et al. (1992).
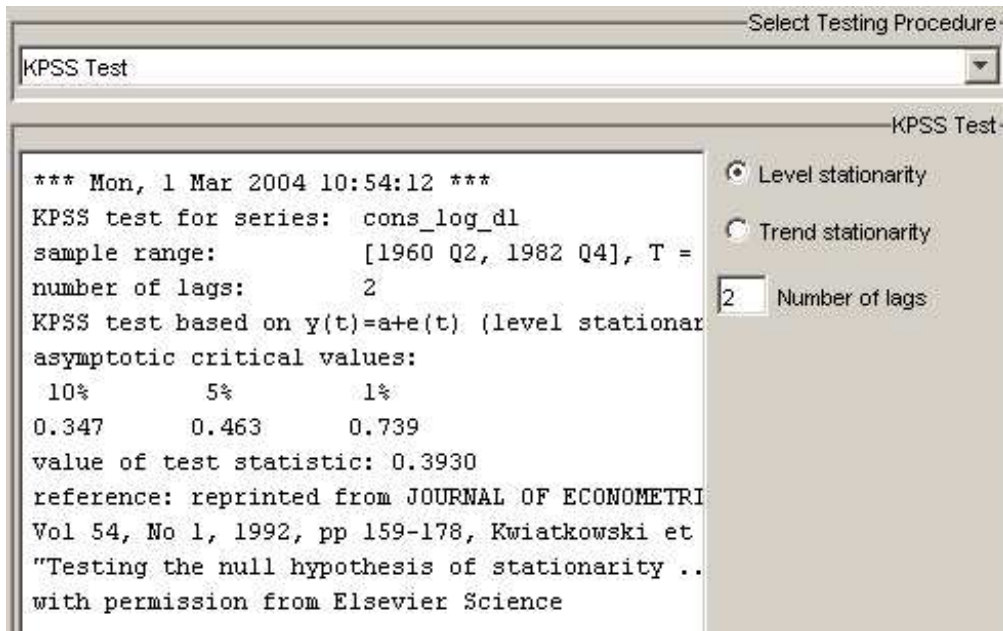
Figure 8: Specification of the KPSS Test

10.2 Specification in JMulTi

selection of variables   one endogenous variable may be selected, exogenous or user defined deterministic variables are ignored

statistics   One may choose between the two possible statistics and adjust the number of lags for the computation of the so-called long-run variance. Suitable choices of the lag length l_q may be $l_4 \approx 4(T/100)^{1/4}$ or $l_{12} \approx 12(T/100)^{1/4}$.

11 Testing for Seasonal Unit Roots

11.1 Background

Tests have been proposed by Hylleberg, Engle, Granger and Yoo (1990) to check for seasonal unit roots in *quarterly* time series. They are based on the model

$$\Delta_4 y_t = \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_3 z_{3,t-1} + \pi_4 z_{3,t-2} + \sum_{j=1}^{p} \alpha_j^* \Delta_4 y_{t-j} + u_t, \tag{3}$$

where $z_{1t} = (1 + L + L^2 + L^3)y_t$, $z_{2t} = -(1 - L + L^2 - L^3)y_t$ and $z_{3t} = -(1 - L^2)y_t$ with L being the lag operator. The null hypotheses $H_0 : \pi_1 = 0$, $H_0 : \pi_2 = 0$ and $H_0 : \pi_3 = \pi_4 = 0$ correspond to tests for regular, semiannual and annual unit roots, respectively. These hypotheses can be tested by estimating the model (3) by OLS and considering the relevant 't-' and 'F-tests'. These tests are known as HEGY tests. Franses and Hobijn (1997) provide suitable critical values which are used in JMulTi . 'F-tests' may also be used for the joint null hypothesis that π_2, π_3 and π_4 are all zero and that all four π's are jointly zero ($\pi_1 = \pi_2 = \pi_3 = \pi_4 = 0$). The asymptotic distributions of the test statistics under the respective null hypotheses depend on the deterministic terms in the model.

The number of lagged seasonal differences $\Delta_4 y_{t-j}$ has to be chosen before the HEGY tests can be performed. This may again be done by using model selection criteria or parameter significance tests.

For *monthly* series the corresponding tests for seasonal unit roots were discussed by Franses (1990) based on the model

$$\begin{aligned} \Delta_{12} y_t = \ & \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_3 z_{3,t-1} + \pi_4 z_{3,t-2} \\ & + \pi_5 z_{4,t-1} + \pi_6 z_{4,t-2} + \pi_7 z_{5,t-1} + \pi_8 z_{5,t-2} \\ & + \pi_9 z_{6,t-1} + \pi_{10} z_{6,t-2} + \pi_{11} z_{7,t-1} + \pi_{12} z_{7,t-2} \\ & + \sum_{j=1}^{p} \alpha_j^* \Delta_{12} y_{t-j} + u_t, \end{aligned} \tag{4}$$

where

$$\begin{aligned} z_{1,t} &= (1 + L)(1 + L^2)(1 + L^4 + L^8)y_t \\ z_{2,t} &= -(1 - L)(1 + L^2)(1 + L^4 + L^8)y_t \\ z_{3,t} &= -(1 - L^2)(1 + L^4 + L^8)y_t \\ z_{4,t} &= -(1 - L^4)(1 - \sqrt{3}L + L^2)(1 + L^4 + L^8)y_t \\ z_{5,t} &= -(1 - L^4)(1 + \sqrt{3}L + L^2)(1 + L^4 + L^8)y_t \\ z_{6,t} &= -(1 - L^4)(1 - L^2 + L^4)(1 - L + L^2)y_t \\ z_{7,t} &= -(1 - L^4)(1 - L^2 + L^4)(1 + L + L^2)y_t. \end{aligned}$$

The process y_t has a regular (zero frequency) unit root if $\pi_1 = 0$ and it has seasonal unit roots if any one of the other π_i $(i = 2, \ldots, 12)$ is zero. For the conjugate complex roots, $\pi_i = \pi_{i+1} = 0$ $(i = 3, 5, 7, 9, 11)$ is required. The corresponding statistical hypotheses can again be checked by t- and F-statistics, critical values for which are given by Franses and

Hobijn (1997). If all the π_i $(i = 1, \ldots, 12)$ are zero, then a stationary model for the monthly seasonal differences of the series is suitable. As in the case of quarterly series it is also possible to include deterministic terms in the model (4).
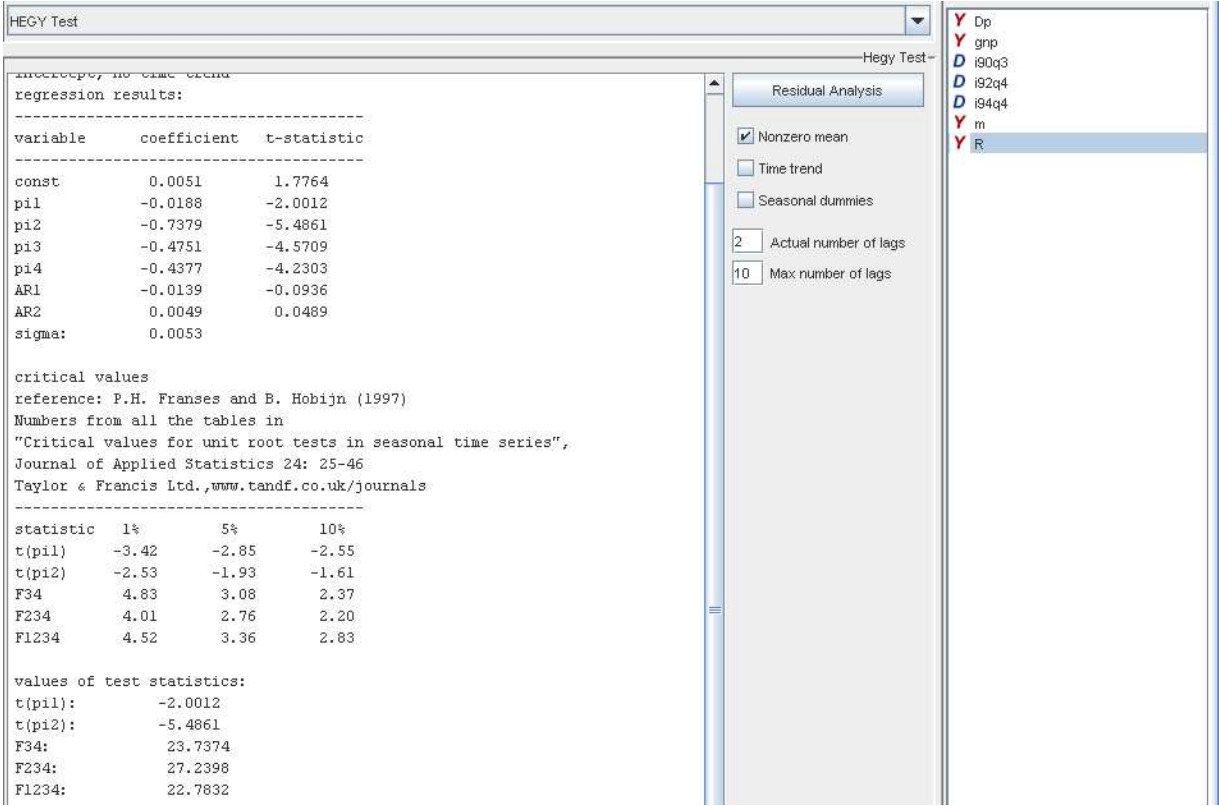
11.2 Specification in JMulTi

Figure 9: Specification of HEGY Test

Input

selection of variables One endogenous variable may be selected, exogenous or user defined deterministic variables are ignored. The selected time series must have periodicity 4 or 12. For other periodicities the test is not implemented in JMulTi .

nonzero mean constant added to regression

time trend constant and trend term added to regression

seasonal dummies seasonal dummies and constant added to regression

actual number of lags the number of lagged differences in the regression

max number of lags   the number of lagged differences taken into account for the computation of the information criteria, max lag does not affect the sample for the test regression, see also Sec. 14

residual analysis   see Sec. 13

Output

pi 1,...,periodicity   the results from the test regression (3) or (4)

AR 1,...,lags   the coefficients of $\Delta_4 y_{t-i}$ $(\Delta_{12} y_{t-i})$

sdummy(i)   the ith seasonal dummy

sigma   the standard deviation $\sqrt{\hat{u}'\hat{u}/(T-K)}$, with K being the number of regressors

test statistics   the respective F- and t-statistics along with the critical values, the pi($= \pi$) coefficients tested are indicated with an index:

- F1112 tests H_0 : pi11=pi12=0

- F2-12 tests H_0 : pi2=pi3=...=pi12=0

- t(pi1) tests H_0 : pi1=0 , etc.

information criteria   the optimal number of lagged differences found by the respective criterion are presented, see Sec. 14

residual analysis   see Sec. 13

12 Tests for Processes with Level Shifts

12.1 Background

If there is a shift in the level of the DGP, it should be taken into account in testing for a unit root because the ADF test may be distorted if the shift is simply ignored. Therefore a shift function, which is here denoted by $f_t(\theta)'\gamma$, may be added to the deterministic term μ_t of the DGP. Hence, a model

$$y_t = \mu_0 + \mu_1 t + f_t(\theta)'\gamma + x_t, \tag{5}$$

is considered, where θ and γ are unknown parameters or parameter vectors and the errors x_t are generated by an $AR(p)$ process with possible unit root.

JMulTi offers the following three possible shift functions:

1. A simple shift dummy variable with shift date T_B,

$$f_t^{(1)} = d_{1t} := \begin{cases} 0, & t < T_B \\ 1, & t \geq T_B \end{cases}.$$

   The function does not involve any extra parameter θ. In the shift term $f_t^{(1)}\gamma$, the parameter γ is a scalar. Differencing this shift function leads to an impulse dummy.

2. The second shift function is based on the exponential distribution function which allows for a nonlinear gradual shift to a new level starting at time T_B,

$$f_t^{(2)}(\theta) = \begin{cases} 0, & t < T_B \\ 1 - \exp\{-\theta(t - T_B + 1)\}, & t \geq T_B \end{cases}.$$

   In the shift term $f_t^{(2)}(\theta)\gamma$, both θ and γ are scalar parameters. The first one is confined to the positive real line $(\theta > 0)$, whereas the second one may assume any value.

3. The third function can be viewed as a rational function in the lag operator applied to a shift dummy d_{1t},

$$f_t^{(3)}(\theta) = \left[\frac{d_{1,t}}{1 - \theta L} : \frac{d_{1,t-1}}{1 - \theta L} \right]'.$$

   Here the actual shift term is $[\gamma_1(1 - \theta L)^{-1} + \gamma_2(1 - \theta L)^{-1}L]d_{1t}$, where θ is a scalar parameter between 0 and 1 and $\gamma = (\gamma_1 : \gamma_2)'$ is a two-dimensional parameter vector.

Both $f_t^{(2)}(\theta)\gamma$ and $f_t^{(3)}(\theta)'\gamma$ can generate sharp one-time shifts at time T_B for suitable values of θ. Thus they are more general than $f_t^{(1)}\gamma$.

Saikkonen and Lütkepohl (2002) and Lanne, Lütkepohl and Saikkonen (2002) propose unit root tests for the model (5) which are based on estimating the deterministic term first by a generalized least squares (GLS) procedure under the unit root null hypothesis and

subtracting it from the original series. Then an ADF type test is performed on the adjusted series which also includes terms to correct for estimation errors in the parameters of the deterministic part. As in the case of the ADF statistic, the asymptotic null distribution is nonstandard. Critical values are tabulated in Lanne et al. (2002). Seasonal dummies may be included in addition to a constant or a linear trend term.

The user of the test has to decide on the AR order and the shift date T_B. If the latter quantity is known, the desired shift function may be included and the AR order may be chosen in the usual way with the help of order selection criteria, sequential tests and model checking tools. If the break date is unknown, Lanne, Lütkepohl and Saikkonen (2001) recommend to choose a reasonably large AR order in a first step and then pick the break date which minimizes the GLS objective function used to estimate the parameters of the deterministic part. In this step, JMulTi uses a shift dummy as shift function. The program offers graphs which show the impact of changes in the break date and variations in the parameter θ. Once a possible break date is fixed, a more detailed analysis of the AR order may be useful to improve the power of the test.
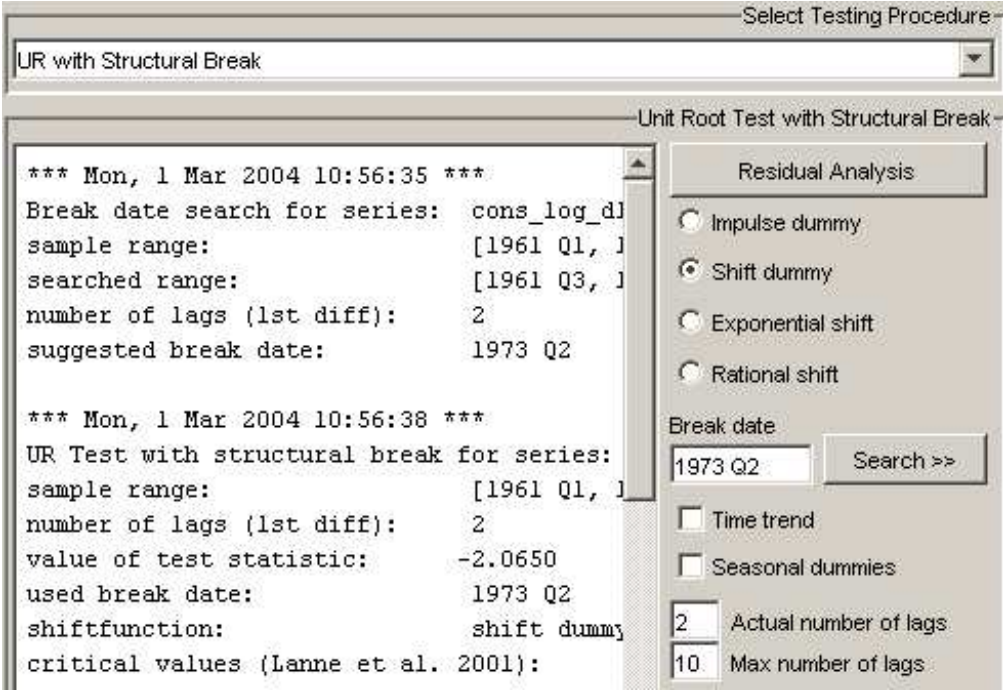
12.2   Specification in JMulTi

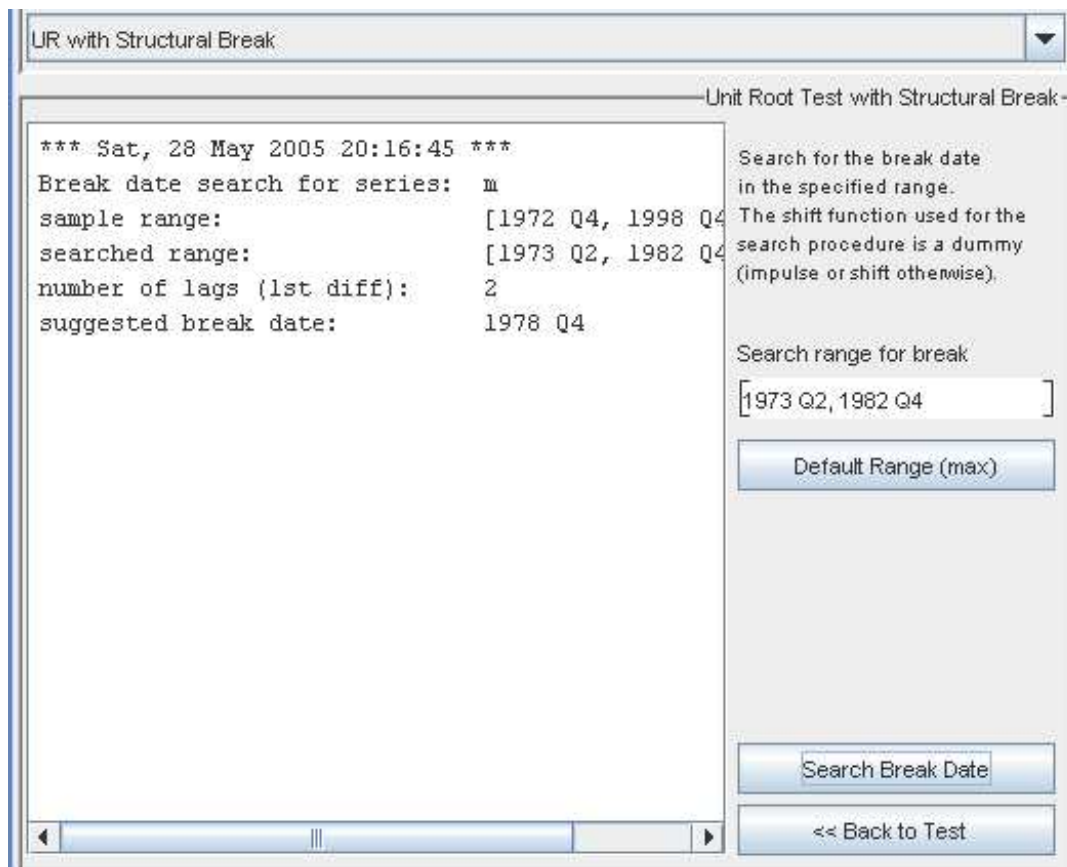

Figure 10: UR Test with Level Shift

Figure 11: automatic break search

Input

selection of variables   one endogenous variable may be selected, exogenous or user defined deterministic variables are ignored, a constant is always automatically added to the regression.

shift function   select the desired shift function, an impulse dummy can be used with 1st differences, for the exponential and rational shift function, the parameter theta($= \theta$) is estimated by a grid search

break date   the desired break date to be used in the regression, click on search to use an automatic procedure to find the optimal break date

time trend   constant and trend term added to regression

seasonal dummies   seasonal dummies and constant added to regression

actual number of lags   the number of lagged differences in the regression

max number of lags the number of lagged differences taken into account for the computation of the information criteria, max lag does not affect the sample period for the test regression, see also Sec. 14

residual analysis see Sec. 13

graphical analysis shows plots of the shift function, the original series with and without deterministic terms and the grid search for theta

Output

value of test statistic the t-statistic of the relevant parameter in the ADF type model setup proposed by Lanne et al. (2002)

theta in case of exponential or rational shift function the estimated slope parameter

dx(-i) stands for Δy_{t-i}

d(const) stands for $Z_1 = [1, 0, \ldots, 0]'$, regressor for initial estimation of the constant

d(trend) stands for $Z_2 = [1, 1, \ldots, 1]'$, regressor for initial estimation of the trend

d(shiftfkt) stands for $Z_3 = [f_1(\theta) : \Delta f_2(\theta) : \cdots : \Delta f_T(\theta)]'$, regressor for initial estimation of the shift parameter γ

d(SD) stands for the seasonal dummies which are included in the same way as Z_i

RSS residual sum of squares $\sum_{t=1}^{T} \hat{u}_t^2$

information criteria the optimal number of lagged differences for the respective criteria are presented, see Sec. 14

residual analysis see Sec. 13

13 Residual Analysis
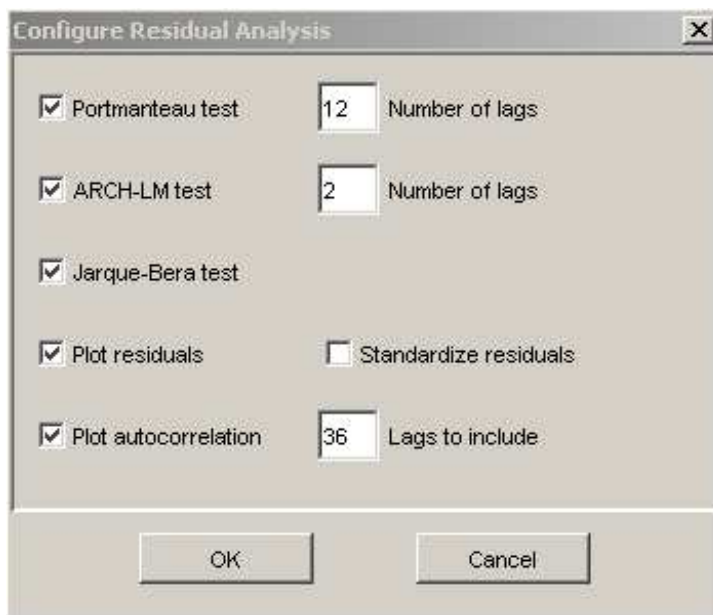
13.1 Specification in JMulTi

Figure 12: Specification of Residual Analysis

The configuration panel allows to specify a range of diagnostic tests for the residual analysis of the estimated model. Select the appropriate checkboxes and adjust the available parameters according to your needs. The residual analysis is then carried out after the respective model is estimated.

13.2 Background

In the following the residual series are denoted by $\hat{u}_t$ $(t = 1, \ldots, T)$ and the standardized residuals are obtained by dividing by the standard deviation, that is, the standardized residuals are $\hat{u}_t^s = (\hat{u}_t - \bar{\hat{u}})/\tilde{\sigma}_u$, where $\tilde{\sigma}_u^2 = T^{-1} \sum_{t=1}^{T} (\hat{u}_t - \bar{\hat{u}})^2$ with $\bar{\hat{u}} = T^{-1} \sum_{t=1}^{T} \hat{u}_t$.

Portmanteau Test

The pair of hypotheses

$$H_0 : \rho_{u,1} = \cdots = \rho_{u,h} = 0 \quad \text{vs.} \quad H_1 : \rho_{u,i} \neq 0 \text{ for at least one } i = 1, \ldots, h,$$

is tested. Here $\rho_{u,i} = \mathsf{Corr}(u_t, u_{t-i})$ denotes the AC coefficients of the residual series. Two test statistics, Q_h and LB_h, are given in JMulTi together with the corresponding p-values

(see Ljung and Box (1978)):

$$Q_h = T \sum_{j=1}^{h} \hat{\rho}_{u,j}^2 \quad \text{and} \quad LB_h = T^2 \sum_{j=1}^{h} \frac{1}{T-j} \hat{\rho}_{u,j}^2$$

where $\hat{\rho}_{u,j} = T^{-1} \sum_{t=j+1}^{T} \hat{u}_t^s \hat{u}_{t-j}^s$. If the $\hat{u}_t$ are residuals from an estimated ARMA(p,q) model, the test statistics have an approximate asymptotic $\chi^2(h-p-q)$ distribution if the null hypothesis holds.

ARCH-LM Test

In JMulTi the test for neglected conditional heteroskedasticity (ARCH) is based on fitting an ARCH(q) model to the estimation residuals,

$$\hat{u}_t^2 = \beta_0 + \beta_1 \hat{u}_{t-1}^2 + \cdots + \beta_q \hat{u}_{t-q}^2 + error_t, \tag{6}$$

and checking the null hypothesis

$$H_0 : \beta_1 = \cdots = \beta_q = 0 \quad \text{vs.} \quad H_1 : \beta_1 \neq 0 \text{ or } \ldots \text{ or } \beta_q \neq 0.$$

Under normality assumptions the LM test statistic is obtained from the coefficient of determination, R^2, of the regression (6):

$$ARCH_{LM}(q) = TR^2.$$

It has an asymptotic $\chi^2(q)$ distribution if the null hypothesis of no conditional heteroskedasticity holds (Engle (1982)).

Jarque-Bera Test

Lomnicki (1961) and Jarque and Bera (1987) have proposed a test for nonnormality based on the skewness and kurtosis of a distribution. The test checks the pair of hypotheses

$$H_0 : \mathsf{E}(u_t^s)^3 = 0 \text{ and } \mathsf{E}(u_t^s)^4 = 3 \quad \text{vs.} \quad H_1 : \mathsf{E}(u_t^s)^3 \neq 0 \text{ or } \mathsf{E}(u_t^s)^4 \neq 3.$$

The test statistic is

$$JB = \frac{T}{6} \left[T^{-1} \sum_{t=1}^{T} (\hat{u}_t^s)^3 \right]^2 + \frac{T}{24} \left[T^{-1} \sum_{t=1}^{T} (\hat{u}_t^s)^4 - 3 \right]^2,$$

and has an asymptotic $\chi^2(2)$ distribution if the null hypothesis is correct (see Jarque and Bera (1987)). In JMulTi also skewness and kurtosis of the standardized residuals are given.

Plot Residuals

Absolute or standardized residuals are plotted.

Plot Autocorrelation

In JMulTi residual autocorrelations (ACs) $\tilde{\rho}_{u,h} = \tilde{\gamma}_{u,h}/\tilde{\gamma}_{u,0}$ are obtained from

$$\tilde{\gamma}_h = \frac{1}{T} \sum_{t=h+1}^{T} (\hat{u}_t - \bar{\hat{u}})(\hat{u}_{t-h} - \bar{\hat{u}})$$

where $\bar{\hat{u}} = T^{-1} \sum_{t=1}^{T} \hat{u}_t$ is the sample mean.

The partial autocorrelation (PAC) between u_t and u_{t-h} is the conditional autocorrelation given $u_{t-1}, \ldots, u_{t-h+1}$. The corresponding sample quantity $\hat{a}_h$ is obtained as the OLS estimator of the coefficient α_h in an autoregressive model

$$\hat{u}_t = \nu + \alpha_1 \hat{u}_{t-1} + \cdots + \alpha_h \hat{u}_{t-h} + error_t.$$

In JMulTi , OLS estimates are obtained for each h with sample size $T - h$.

14 Information Criteria

The following formulas for the information criteria are used:

$$\text{AIC}(n) = \log \widetilde{\sigma}_u^2(n) + \frac{2}{T}n \qquad (\text{Akaike (1973, 1974)}) \ ,$$

$$\text{HQ}(n) = \log \widetilde{\sigma}_u^2(n) + \frac{2 \log \log T}{T}n \qquad (\text{Hannan and Quinn (1979)}),$$

$$\text{SC}(n) = \log \widetilde{\sigma}_u^2(n) + \frac{\log T}{T}n \qquad (\text{Schwarz (1978) and Rissanen (1978)}),$$

and

$$\text{FPE}(n) = \frac{T + n^*}{T - n^*}\widetilde{\sigma}_u^2(n) \qquad (\text{Akaike (1969)}),$$

where $\widetilde{\sigma}_u^2(n)$ is estimated by $\hat{u}'\hat{u}/T$, n is the number of lagged differences included and n^* is the total number of parameters in the model when n lagged differences are included.
The computation of the $\hat{u}_t$ is always done with a regression model that is estimated with OLS. All models with 0 to n lagged differences are estimated. The lag length which minimizes the respective information criterion is presented. The sample length is the same for all different lag lengths and is determined by the maximum order. In other words, the number of values set aside as presample values is determined by the maximum lag order.

15 Cointegration Tests

The cointegration tests in JMulTi are based on the following general model

$$y_t = D_t + x_t$$

where y_t is a K-dimensional vector of observable variables, D_t is a deterministic term, e.g., $D_t = \mu_0 + \mu_1 t$ may be a linear trend term, and x_t is a VAR(p) process with vector error correction model (VECM) representation

$$\Delta x_t = \Pi x_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_t$$

Here u_t is a vector white noise process with $u_t \sim (0, \Sigma_u)$. The rank of Π is the cointegrating rank of x_t and hence of y_t. Therefore the cointegration tests check the pair of hypotheses

$$H_0(r_0) : \text{rk}(\Pi) = r_0 \quad \text{versus} \quad H_1(r_0) : \text{rk}(\Pi) > r_0, \qquad r_0 = 0, \ldots, K-1 \qquad (7)$$

Two types of tests are available in JMulTi , Johansen trace tests and tests proposed by Saikkonen & Lütkepohl. For both types of tests the VAR order p has to be specified. Model selection criteria offer help in the decision on the VAR order (see Sec. 18).

16 Johansen Trace Tests

16.1 Background

Johansen (1988, 1991, 1992, 1994, 1995) in a series of publications has proposed tests which are likelihood ratio (LR) tests if y_t is normally distributed and Gaussian pseudo LR tests otherwise. In the literature these tests are known as *trace tests* because of the special form of the test statistic. The distributions of the test statistics under their respective null hypotheses depend on the deterministic terms. In JMulTi three basic modelling options are available where in each case seasonal dummy variables and impulse dummies may be added. Furthermore, in case 1 and 2 the user may specify up to two structural breaks which appear either in levels only or in trend and levels jointly (only case 2).

Eviews Compatibility Guide

Users with experience in the econometric software package Eviews should note that in JMulTi only three relevant test cases are implemented, which are described in the following sections. The mapping between the Eviews and JMulTi model specification options is the following (Eviews - JMulTi):

- no intercept in CE or VAR - not implemented

- intercept in CE, no intercept in VAR - constant (case 1)

- intercept in CE and test VAR - orthogonal trend (case 3)

- intercept and trend in CE, no trend in VAR - constant and trend (case 2)

- intercept and trend in CE, linear trend in VAR - not implemented

Case 1: Restricted mean term and no linear trend

In this case the deterministic term has the form

$$D_t = \mu_0 (+\text{seasonal dummies})$$

and the DGP of the y_t can be written as

$$\Delta y_t = \Pi^* \begin{bmatrix} y_{t-1} \\ 1 \end{bmatrix} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_t$$

where $\Pi^* = [\Pi : \nu_0]$ is $(K \times (K+1))$ with $\nu_0 = -\Pi\mu_0$ and the seasonals are neglected. The test statistic is obtained by reduced rank regression applied to this model with $\text{rk}(\Pi^*) = r_0$ (see Johansen (1995)).

Case 2: Constant and linear trend

In this case the deterministic term has the form

$$D_t = \mu_0 + \mu_1 t (+\text{seasonal dummies})$$

and the DGP of the y_t can be written as

$$\Delta y_t = \nu + \Pi^+ \begin{bmatrix} y_{t-1} \\ t-1 \end{bmatrix} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_t$$

where $\Pi^+ = \alpha[\beta' : \eta]$ is a $(K \times (K+1))$ matrix of rank r_0 with $\eta = -\beta' \mu_1$ and the seasonals are neglected. The test is based on this model (see Johansen (1994, 1995)).

Case 3: Trend orthogonal to cointegration relations

In this case the deterministic term again has the form

$$D_t = \mu_0 + \mu_1 t (+\text{seasonal dummies})$$

It is assumed, however, that there is a linear trend term in the variables but not in the cointegration relations (it is orthogonal to the cointegration relations) so that $\Pi(y_{t-1} - \mu_0 - \mu_1(t-1)) = \Pi(y_{t-1} - \mu_0)$. In this case the model for y_t can be written as

$$\Delta y_t = \nu + \Pi y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_t$$

(see Johansen (1995)). In this setup it is not meaningful to test $H_0 : \text{rk}(\Pi) = K - 1$ versus $H_1 : \text{rk}(\Pi) = K$, as argued by Saikkonen and Lütkepohl (2000a).

16.2   Specification in JMulTi

Input

selecting variables   a number endogenous and deterministic variables can freely be selected for this test, but deterministic variables must not be a shift or a trend shift because this should be specified via the *Set Breaks* dialog

structural breaks   For a discussion of the test specification in case of structural breaks, see Johansen, Mosconi and Nielsen (2000). Trenkler (2004) discusses the case where a trend model is specified but only a break in levels occurs.
The *Set Breaks* dialog can be used to specify up to two structural breaks either in levels only or in levels and trend jointly. In case 1 only level breaks are used, even if trend breaks are selected. In case 2 the breaks might occur in trend and levels jointly or in levels only. In case 3 all structural breaks are ignored because for this critical values are not easily obtained.
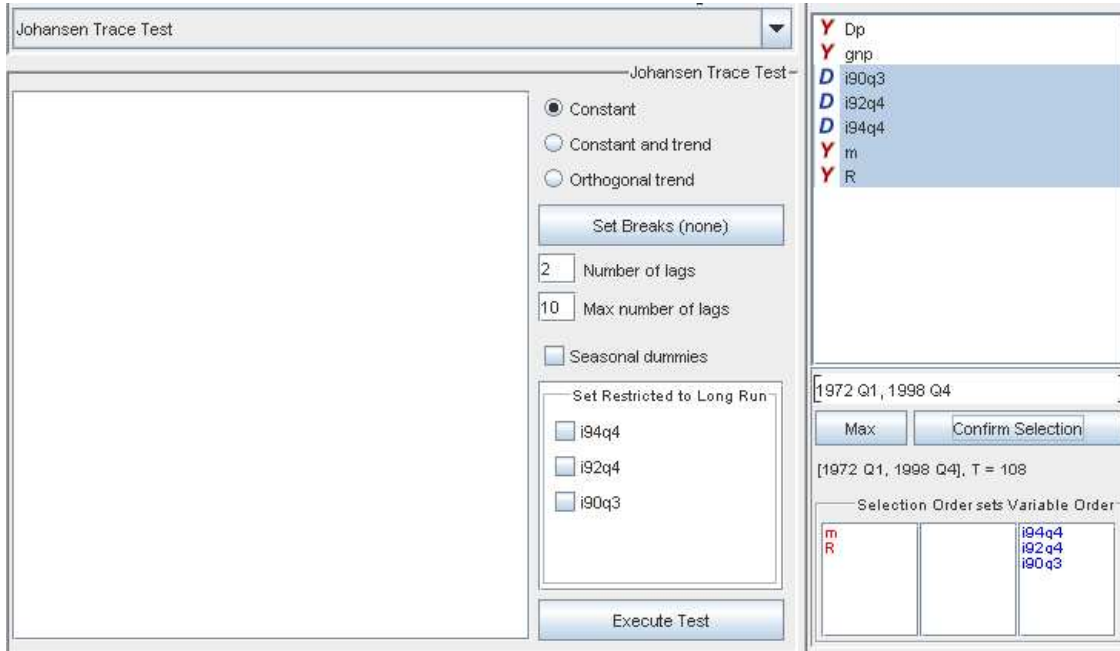
Figure 13: Specification of Johansen Trace Test

If structural breaks are specified then at the beginning of each subsample p dummies are included in the model to condition on these observations. This is done automatically by the program. The shift dummies enter the model in the following way:

- case 1, break in levels - level shift(s) restricted to EC term

- case 2, break in levels only - level shift(s) restricted to EC term,

- case 2, break in levels and trend jointly - trend shift(s) restricted to EC term and level shift(s) unrestricted in model

nonzero mean   this corresponds to case 1

constant and trend   this corresponds to case 2

orthogonal trend   this corresponds to case 3

seasonal dummies   centered seasonal dummies are added to the model, centered means that $SD_{1t} + ... + SD_{Pt} = 0$ for each t, where P is the periodicity of the data

actual number of lags   the VAR order p of the levels VAR form

max number of lags   maximum VAR order considered by the model selection criteria, see Sec. 18

Figure 14: Specification of Level and Trend Breaks

set restricted to long run   If additional impulse dummies terms are included they appear in a list. By selecting them one can create a model of the form

$$\Delta y_t = \Pi \begin{bmatrix} y_{t-1} \\ Dl_{t-1} \end{bmatrix} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + u_t$$

with Dl_t being the vector of deterministic terms restricted to the EC term.

Output

r0   the tested rank of the matrix Π under H_0, $r0 = 0, ..., K-1$, in case of an orthogonal trend $r0 = 0, ..., K-2$, see Sec. 16.1

critical values and p-values   The critical values as well as the p-values of all Johansen trace tests are obtained by computing the respective response surface according to Doornik (1998) if there are no breaks, or according to Johansen et al. (2000) if there are up to 2 breaks. In case 2 with a break only in the levels the response surface also follows the design of Johansen et al. (2000). However, the simulation of the limiting distribution is based on a representation as in Theorem 3.1 for the case of no trend breaks.

optimal number of lags   the optimal VAR order determined by the information criteria

17   Saikkonen & Lütkepohl Tests

17.1   Background

Saikkonen and Lütkepohl (2000a, b,c) have proposed tests for the pair of hypotheses (7) which proceed by estimating the deterministic term D_t first, subtracting it from the observations and applying a Johansen type test to the adjusted series. In other words, the test is based on a reduced rank regression of the system

$$\Delta\tilde{x}_t = \Pi\tilde{x}_{t-1} + \sum_{j=1}^{p-1}\Gamma_j\Delta\tilde{x}_{t-j} + \tilde{u}_t$$

where $\tilde{x}_t = y_t - \tilde{D}_t$ and $\tilde{D}_t$ is the estimated deterministic term. The parameters of the deterministic term are estimated by the GLS procedure proposed by Saikkonen and Lütkepohl. The critical values depend on the kind of deterministic term included. Possible options are a constant, a linear trend term, a linear trend orthogonal to the cointeration relations and seasonal dummy variables. In other words, all the options available for the Johansen trace tests are also available here. In addition, the critical values remain valid if a shift dummy variable is included. However, trend breaks are ignored by this test.
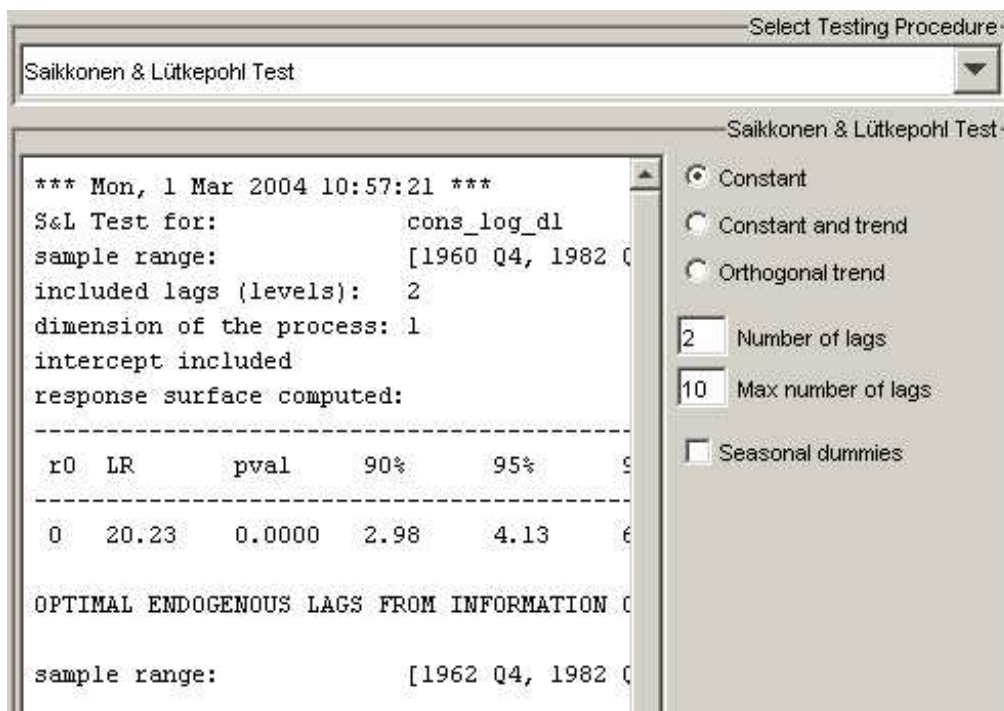
17.2   Specification in JMulTi

Figure 15: Specification of S&L Test

Input

selecting variables   a number endogenous and deterministic variables can freely be selected for this test, but deterministic variables that are trend shifts will be ignored

nonzero mean   this corresponds to case 1, see Sec. 16.1

constant and trend   this corresponds to case 2, see Sec. 16.1

orthogonal trend   this corresponds to case 3, see Sec. 16.1

seasonal dummies   centered seasonal dummies are added to the model, centered means that $SD_{1t} + ... + SD_{Pt} = 0$ for each t, where P is the periodicity of the data

actual number of lags   the VAR order p of the levels VAR form

max number of lags   maximum VAR order considered by the model selection criteria, see Sec. 18

Output

r0   the tested rank of the matrix Π under H_0, $r0 = 0, ..., K - 1$, in case of an orthogonal trend $r0 = 0, ..., K - 2$, see Sec. 16.1

critical values and p-values   response surface generated according to Trenkler (2004)

optimal number of lags   optimal VAR order determined by the information criteria

18 Information Criteria for Vector Processes

The information criteria are computed for VAR models in the levels of the variables,

$$y_t = D_t + A_1 y_{t-1} + \cdots + A_n y_{t-n} + u_t$$

using LS estimation. Here D_t denotes again deterministic terms which are also estimated. The following formulas for the information criteria are used:

$$\text{AIC}(n) = \log \det(\widetilde{\Sigma}_u(n)) + \frac{2}{T} n K^2 \qquad \text{(Akaike (1973, 1974))} ,$$

$$\text{HQ}(n) = \log \det(\widetilde{\Sigma}_u(n)) + \frac{2 \log \log T}{T} n K^2 \qquad \text{(Hannan and Quinn (1979), Quinn (1980))},$$

$$\text{SC}(n) = \log \det(\widetilde{\Sigma}_u(n)) + \frac{\log T}{T} n K^2 \qquad \text{(Schwarz (1978) and Rissanen (1978))},$$

and

$$\text{FPE}(n) = \left(\frac{T + n^*}{T - n^*} \right)^K \det(\widetilde{\Sigma}_u(n)) \qquad \text{(Akaike (1971))},$$

where $\widetilde{\Sigma}_u(n)$ is estimated by $T^{-1} \sum_{t=1}^{T} \hat{u}_t \hat{u}_t'$, n^* is the total number of parameters in each equation of the model when n is the VAR order, also counting the deterministic terms. The sample length is the same for all different lag lengths and is determined by the maximum order. In other words, the number of values set aside as presample values is determined by the maximum lag order considered. The lag length which minimizes the respective information criterion is presented in the output of JMulTi . For more information on the model selection criteria see Lütkepohl (1991).

In JMulTi the maximum number of lags is automatically reduced by 1 if the regressor matrix has not full column rank after truncating the presample values. This procedure is repeated until a maximum lag is found for which the regression can be executed. If such an adjustment was done, the line *max lag adjusted* is added to the output.

References

Akaike, H. (1969). Fitting autoregressive models for prediction, *Annals of the Institute of Statistical Mathematics* **21**: 243–247.

Akaike, H. (1971). Autoregressive model fitting for control, *Annals of the Institute of Statistical Mathematics* **23**: 163–180.

Akaike, H. (1973). Information theory and an extension of the maximum likelihood principle, *in* B. Petrov and F. Csáki (eds), *2nd International Symposium on Information Theory*, Académiai Kiadó, Budapest, pp. 267–281.

Akaike, H. (1974). A new look at the statistical model identification, *IEEE Transactions on Automatic Control* **AC-19**: 716–723.

Bartlett, M. S. (1950). Periodogram analysis and continuous spectra, *Biometrika* **37**: 1–16.

Davidson, R. and MacKinnon, J. (1993). *Estimation and Inference in Econometrics*, Oxford University Press, London.

Dickey, D. A. and Fuller, W. A. (1979). Estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association* **74**: 427–431.

Doornik, J. A. (1998). Approximations to the asymptotic distributions of cointegration tests, *Journal of Economic Surveys* **12**: 573–593.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity, with estimates of the variance of United Kingdoms inflations, *Econometrica* **50**: 987–1007.

Franses, P. H. (1990). Testing for seasonal unit roots in monthly data, *Econometric Institute Report 9032A*, Erasmus University Rotterdam.

Franses, P. H. and Hobijn, B. (1997). Critical values for unit root tests in seasonal time series, *Journal of Applied Statistics* **24**: 25–46.

Fuller, W. A. (1976). *Introduction to Statistical Time Series*, John Wiley & Sons, New York.

Hannan, E. J. and Quinn, B. G. (1979). The determination of the order of an autoregression, *Journal of the Royal Statistical Society* **B41**: 190–195.

Hylleberg, S., Engle, R. F., Granger, C. W. J. and Yoo, B. S. (1990). Seasonal integration and cointegration, *Journal of Econometrics* **44**: 215–238.

Jarque, C. M. and Bera, A. K. (1987). A test for normality of observations and regression residuals, *International Statistical Review* **55**: 163–172.

Johansen, S. (1988). Statistical analysis of cointegration vectors, *Journal of Economic Dynamics and Control* **12**: 231–254.

Johansen, S. (1991). Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models, *Econometrica* **59**: 1551–1581.

Johansen, S. (1992). Determination of cointegration rank in the presence of a linear trend, *Oxford Bulletin of Economics and Statistics* **54**: 383–397.

Johansen, S. (1994). The role of the constant and linear terms in cointegration analysis of nonstationary time series, *Econometric Reviews* **13**: 205–231.

Johansen, S. (1995). *Likelihood-based Inference in Cointegrated Vector Autoregressive Models*, Oxford University Press, Oxford.

Johansen, S., Mosconi, R. and Nielsen, B. (2000). Cointegration analysis in the presence of structural breaks in the deterministic trend, *Econometrics Journal* **3**: 216–249.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P. and Shin, Y. (1992). Testing the null of stationarity against the alternative of a unit root: How sure are we that the economic time series have a unit root?, *Journal of Econometrics* **54**: 159–178.

Lanne, M., Lütkepohl, H. and Saikkonen, P. (2001). Test procedures for unit roots in time series with level shifts at unknown time, *Discussion paper*, Humboldt-Universität Berlin.

Lanne, M., Lütkepohl, H. and Saikkonen, P. (2002). Comparison of unit root tests for time series with level shifts, *Journal of Time Series Analysis* .

Ljung, G. M. and Box, G. E. P. (1978). On a measure of lack of fit in time-series models, *Biometrika* **65**: 297–303.

Lomnicki, Z. A. (1961). Tests for departure from normality in the case of linear stochastic processes, *Metrika* **4**: 37–62.

Lütkepohl, H. (1991). *Introduction to Multiple Time Series Analysis*, Springer Verlag, Berlin.

Quinn, B. G. (1980). Order determination for a multivariate autoregression, *Journal of the Royal Statistical Society* **B42**: 182–185.

Rissanen, J. (1978). Modeling by shortest data description, *Automatica* **14**: 465–471.

Saikkonen, P. and Lütkepohl, H. (2000a). Testing for the cointegrating rank of a VAR process with an intercept, *Econometric Theory* **16**: 373–406.

Saikkonen, P. and Lütkepohl, H. (2000b). Testing for the cointegrating rank of a VAR process with structural shifts, *Journal of Business & Economic Statistics* **18**: 451–464.

Saikkonen, P. and Lütkepohl, H. (2000c). Trend adjustment prior to testing for the cointegrating rank of a vector autoregressive process, *Journal of Time Series Analysis* **21**: 435–456.

Saikkonen, P. and Lütkepohl, H. (2002). Testing for a unit root in a time series with a level shift at unknown time, *Econometric Theory* **18**: 313–348.

Schmidt, P. and Phillips, P. C. B. (1992). LM tests for a unit root in the presence of deterministic trends, *Oxford Bulletin of Economics and Statistics* **54**: 257–287.

Schwarz, G. (1978). Estimating the dimension of a model, *Annals of Statistics* **6**: 461–464.

Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis*, Chapman & Hall, London.

Trenkler, C. (2004). Determining p-values for systems cointegration tests with a prior adjustment for deterministic terms, mimeo, Humboldt-Universität zu Berlin.