

VAR Analysis in JMulTi

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Finite order VAR models can be specified, estimated, analyzed and used for forecasting in JMulTi . The relevant features will be described in the following.

1 The Basic Model

The basic VAR (vector autoregressive) model allowed for in JMulTi has the form

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + B_0 x_t + \dots + B_q x_{t-q} + C D_t + u_t, \quad (1)$$

where $y_t = (y_{1t}, \dots, y_{Kt})'$ is a vector of K observable endogenous variables, $x_t = (x_{1t}, \dots, x_{Mt})'$ is a vector of M observable exogenous or unmodelled variables, D_t contains all deterministic variables which may consist of a constant, a linear trend, seasonal dummy variables as well as user specified other dummy variables, and u_t is a K -dimensional unobservable zero mean white noise process with positive definite covariance matrix $E(u_t u_t') = \Sigma_u$. The A_i , B_j and C are parameter matrices of suitable dimension.

Various restrictions can be imposed on the parameter matrices. In particular, by imposing zero restrictions, the right-hand side variables may not be the same in all equations. For example, some equations may contain specific dummy or exogenous variables which do not appear in other equations. Notice also that $B_0 = 0$ may be specified if the exogenous variables are to appear in lagged form only.

If no exogenous variables are present, (1) is a standard VAR(p) model with deterministic terms D_t . A univariate AR model is obtained if just one y variable is considered ($K = 1$). Thus the present model framework can also be used for univariate or single equation analysis. The AR or VAR order p may be chosen with the help of model selection criteria (see Sec. 3.1).

1.1 General Remark about the Implementation in JMulTi

VAR modeling in JMulTi is meant as a step by step procedure, where each task is related to a special panel. Once a model has been estimated, the diagnostic tests as well as the stability analysis and the structural analysis use the results from the estimation. If changes in the model specification are made by the user, these results are deleted and the model has to be reestimated. In other words, only results related to one model at a time are kept in the system. Hence, there should not be confusion regarding the model setup while going through the analysis. Sometimes certain menus or options are not available. This always has a specific reason which is described in the respective help topic.

2 Estimation

2.1 Background

Estimation of the model (1) is done by feasible generalized least squares (GLS). For this purpose the individual equations of the system are first estimated by OLS. The residuals are used to estimate the white noise covariance matrix Σ_u as $\hat{\Sigma}_u = T^{-1} \sum_{t=1}^T \hat{u}_t \hat{u}_t'$. This estimator is then used in the next step to compute the GLS estimator. If all regressors in all equations are identical, the estimator reduces to an equation by equation OLS estimator.

2.2 Estimation Results in JMulTi

After the model is specified, see Sec. 3, it can be estimated by selecting the Menu *Estimation* \rightarrow *Estimated Model*. Output is generated in matrix and text form. Some statistics related to the model are provided under *Stats*.

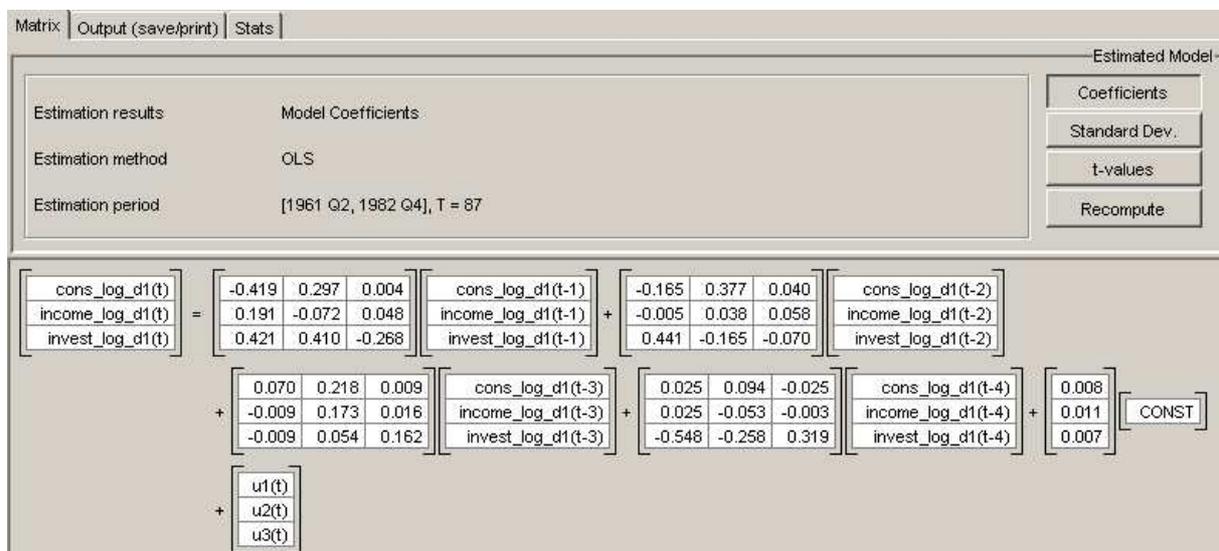


Figure 1: Estimation Results in Matrix Form

The matrix panel displays first the endogenous, then exogenous and finally the deterministic coefficients. It reflects the mathematical notation to make clear, what type of model was actually estimated. By RIGHT clicking on the coefficients tables one can increase or decrease the precision of the numbers. By clicking on the respective buttons it is possible to display either the estimated coefficients, the standard deviations or the t-values.

The text panel displays the same information as the matrix panel but in a form that can be saved as a text file by RIGHT clicking over the text area. In addition to that, it gives the

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Matrix Output (save/print) Stats
*** Mon, 1 Mar 2004 11:11:47 ***
VAR ESTIMATION RESULTS
endogenous variables:   cons_log_dl income_log_dl invest_log_dl
exogenous variables:
deterministic variables:  CONST
endogenous lags:        4
exogenous lags:         0
sample range:           [1961 Q2, 1982 Q4], T = 87

modulus of the eigenvalues of the reverse characteristic polynomial :
|z| = ( 1.2050    1.2050    1.3614    1.5268    1.5268    1.7861    1.7861

Legend:
=====
                Equation 1   Equation 2   ...
-----
Variable 1 | Coefficient           ...
            | (Std. Dev.)
            | (p - Value)
            | [t - Value]
Variable 2 |           ...
...

```

Figure 2: Estimation Results in Text Form

modulus of the eigenvalues of the reverse characteristic polynomial, which is defined as:

$$\det(I_K - A_1z - \dots - A_pz^p), \quad \text{see Lütkepohl (1991), Ch. 2.}$$

3 Model Specification

Specifying a model of the form (1) in JMulTi involves the specification of a maximum lag order for the endogenous and exogenous variables as well as placing zero restrictions on the parameter matrices. While the user is expected to provide a maximum lag order for the exogenous variables, model selection criteria are available to aid in the choice of the VAR order p . Moreover a range of different procedures for imposing zero restrictions on the parameter matrices are offered.

3.1 Model Selection Criteria

3.1.1 Background

The information criteria are computed for VAR models of the form

$$y_t = A_1 y_{t-1} + \dots + A_n y_{t-n} + B_0 x_t + \dots + B_q x_{t-q} + CD_t + u_t,$$

where y_t is K -dimensional. The lag order of the exogenous variables x_t , q , and the deterministic term D_t have to be prespecified by the analyst. For a range of lag orders n the model is estimated by OLS (applied to each equation separately). The optimal lag order is chosen by minimizing one of the following information criteria:

$$\begin{aligned} \text{AIC}(n) &= \log \det(\tilde{\Sigma}_u(n)) + \frac{2}{T} n K^2, \\ \text{HQ}(n) &= \log \det(\tilde{\Sigma}_u(n)) + \frac{2 \log \log T}{T} n K^2, \\ \text{SC}(n) &= \log \det(\tilde{\Sigma}_u(n)) + \frac{\log T}{T} n K^2 \end{aligned}$$

and

$$\text{FPE}(n) = \left(\frac{T + n^*}{T - n^*} \right)^K \det(\tilde{\Sigma}_u(n)),$$

where $\tilde{\Sigma}_u(n)$ is estimated by $T^{-1} \sum_{t=1}^T \hat{u}_t \hat{u}_t'$, n^* is the total number of parameters in each equation of the model when n is the lag order of the endogenous variables, also counting the deterministic terms and exogenous variables. The sample length is the same for all different lag lengths and is determined by the maximum lag order. In other words, the number of values set aside as presample values is determined by the maximum lag order considered for the endogenous and exogenous variables. The lag length which minimizes the respective information criterion is presented in the output of JMulTi. For more information on the model selection criteria see Lütkepohl (1991).

3.1.2 Selection of Variables and Lags in JMulTi

To get to the specification panel you need to select *Specification*→*Specify VAR Model*. To build a model of the form (1) in JMulTi you have to choose the variables you want to include in your model first. By RIGHT clicking on the selected variables you may define exogenous or deterministic variables. The selected user defined variables are shown in their correct order in the available text fields.

You may also adjust the sample by editing the date text fields in the selection panel. To include intercept, trend or seasonal dummies, you should use the available checkboxes. It is possible, however, to add further deterministic variables defined by the user.

To select the endogenous lags it may be helpful to use the information criteria. To do that, choose *Compute Infocriteria*. Then models with the selected variables are estimated. The exogenous lags are taken as given. A search is performed over the lags of the endogenous variables up to the maximum order.

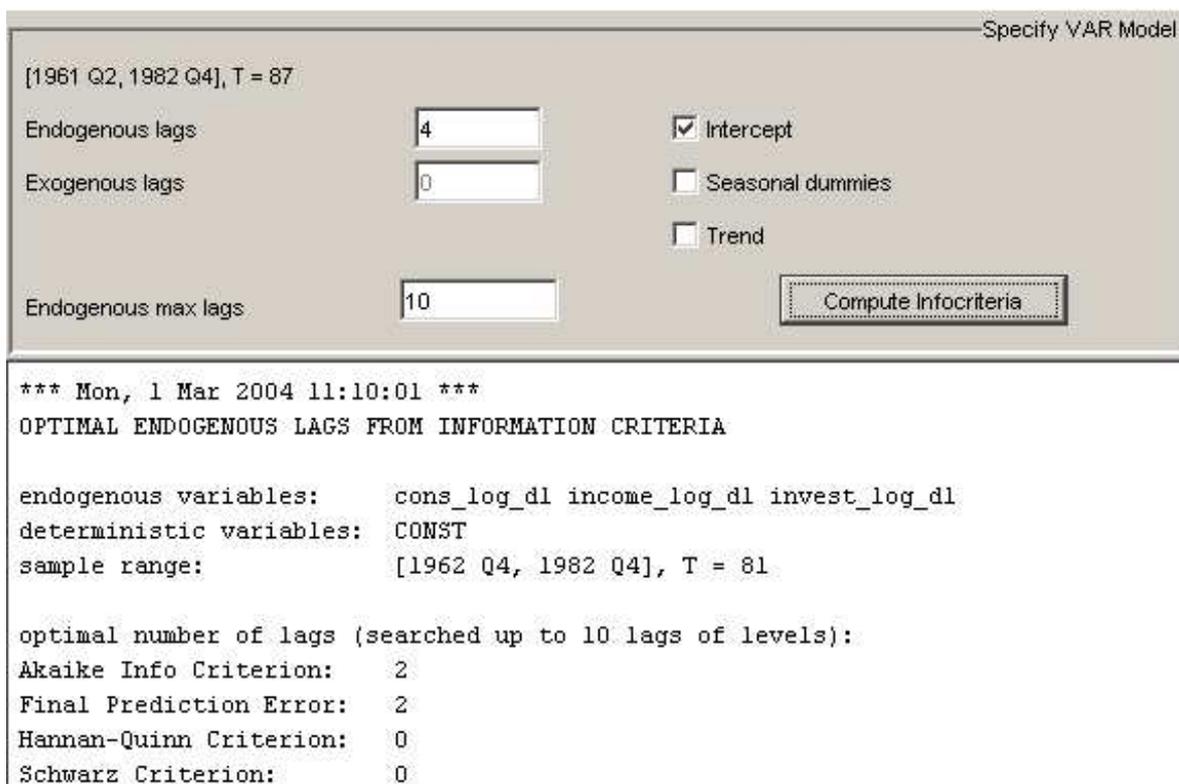


Figure 3: Specification of Variables, Sample Period and Lags

4 Subset Model Selection

4.1 Background

Zero restrictions may be imposed on the parameters of a model based on the t -ratios, for example. Alternatively, restrictions for individual parameters or groups of parameters may be based on model selection criteria. **JMulTi** offers suitable model selection procedures based on single equation methods as well as an algorithm which considers the full system at once in the elimination procedure.

To describe the single equation methods consider the equation

$$y_t = \theta_1 x_{1t} + \dots + \theta_N x_{Nt} + u_t, \quad t = 1, \dots, T. \quad (2)$$

For simplicity, all right-hand side variables are denoted by x_{kt} including exogenous and deterministic variables as well as lagged endogenous variables. The optimal set of regressors is then selected by minimizing a variable selection criterion of the general form

$$CR(i_1, \dots, i_n) = \log(SSE(i_1, \dots, i_n)/T) + c_T n/T, \quad (3)$$

where $SSE(i_1, \dots, i_n)$ is the sum of squared errors obtained by including $x_{i_1t}, \dots, x_{i_nt}$ in the regression model (2) and c_T is a quantity which determines the specific criterion. More precisely,

$$c_T = \begin{cases} 2 & \text{for AIC,} \\ 2 \log \log T & \text{for HQ,} \\ \log T & \text{for SC.} \end{cases}$$

The following **Sequential Elimination of Regressors (SER)** strategy is available in **JMulTi** : Sequentially delete those regressors which lead to the largest reduction of the selected criterion until no further reduction is possible (see, e.g., Brüggemann and Lütkepohl (2001) for more details). This strategy is equivalent to sequentially eliminating those regressors with the smallest absolute values of t -ratios until all t -ratios (in absolute value) are greater than some threshold value. Note that a single regressor is eliminated in each step only. Then new t -ratios are computed for the reduced model.

Another possible sequential elimination algorithm implemented in **JMulTi** is a **Top-Down (TD)** procedure which starts from the last regressor in the equation and checks if deleting it improves the criterion value. In that case it is eliminated. Otherwise it is maintained. Then the second last regressor is checked and so on. Obviously, this procedure depends on the ordering of the variables in the model and, hence, in the equation.

There is also a **System SER** procedure implemented in **JMulTi** . In this procedure, in each step the parameter with the smallest t -ratio is checked and potentially eliminated. The decision regarding the elimination can be based on model selection criteria or a threshold value is specified and only variables with a t -ratio larger than the threshold are maintained eventually.

4.2 Specification of Subset Restrictions and Search Strategy in JMulTi

To get to the subset specification panel you need to select *Specification*→*Subset Restrictions*. There you see the full model defined in the selection panel, see Sec. 3, with the possibility to include or exclude certain coefficients from the estimation. Again you may use the RIGHT mouse click over the tables to set a property for whole matrices.

It is also possible to let JMulTi search for restrictions automatically. To do this you need to define a search strategy. The following strategies are available:

System SER corresponds to 4.1 based on model selection criteria

SER/Testing Procedure see 4.1

Top Down see 4.1

System Testing Procedure corresponds to 4.1 based on a threshold value

For each strategy one may select the model selection criterium, for the **System Testing Procedure** the threshold value can be defined. You may exclude or include certain variables regardless of what the search procedure tells you by setting them either to **0** or to **!**. If you do not use a search procedure but estimate the model with manually set restrictions, there is no difference between **!** and *****. To see the results of the estimation with subset restrictions, See 2.2.

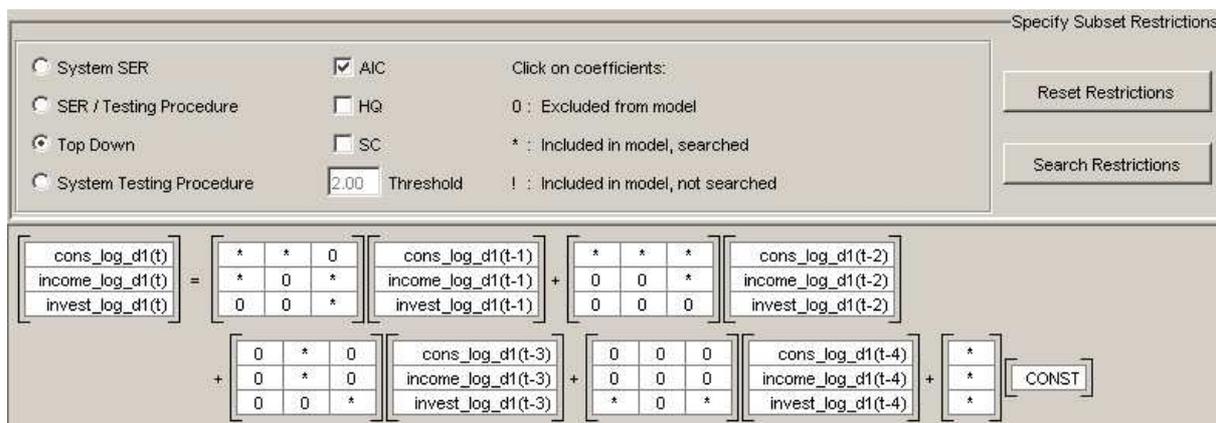


Figure 4: Specification of Subset Restrictions and Search Strategy

5 Residual Analysis

To access the residual analysis you have to specify and estimate a model first. It is then possible to select the menu item *Model Checking*→*Residual Analysis*. In JMulTi the residual analysis is split up into several different panels.



Figure 5: Panels for Residual Analysis

Diagnostic tests to convey a range of diagnostic tests, see Sec. 6

Plot/Add residuals can be plotted in several ways as well as added again to the set of available series

Correlation for graphical autocorrelation and crosscorrelation analysis see Sec. 7

Spectrum it is possible to show the spectrum of the available residuals, see the respective description in helpsection *Initial Analysis*

Kernel Density for a description of kernel density estimation, see the respective description in helpsection *Initial Analysis*

6 Diagnostic Tests

6.1 Background

In JMulTi, tests for residual autocorrelation, nonnormality and conditional heteroskedasticity are available for diagnostic checking of estimated VAR models. It is also possible to show the covariance and correlation matrix together with its determinant and the values of model selection criteria associated with a given model.

Portmanteau test for autocorrelation

A portmanteau test for residual autocorrelation may be applied if a pure VAR process possibly with subset restrictions but without exogenous variables has been fitted. The test checks the null hypothesis

$$H_0 : \mathbb{E}(u_t u'_{t-i}) = 0, \quad i = 1, \dots, h,$$

against the alternative that at least one autocovariance and, hence, one autocorrelation is nonzero. The test statistic has the form

$$Q_h = T \sum_{j=1}^h \text{tr}(\hat{C}'_j \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1})$$

where $\hat{C}_i = T^{-1} \sum_{t=i+1}^T \hat{u}_t \hat{u}'_{t-i}$. If the \hat{u}_t are residuals from a stable VAR(p) process, Q_h has an approximate $\chi^2(K^2 h - n^*)$ distribution under the null hypothesis. Here n^* denotes the number of estimated VAR parameters, not counting the parameters related to the deterministic terms. The limiting χ^2 distribution is strictly valid only if $h \rightarrow \infty$ at a suitable rate with growing sample size. The following adjusted portmanteau statistic is also available,

$$Q_h^* = T^2 \sum_{j=1}^h \frac{1}{T-j} \text{tr}(\hat{C}'_j \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1}).$$

It may have better small sample properties than the unadjusted version.

The choice of h is important for the test performance. If h is chosen too small, the χ^2 approximation to the null distribution may be very poor whereas a large h may result in a loss of power.

Breusch-Godfrey LM test for autocorrelation

The Breusch-Godfrey LM test for h -th order residual autocorrelation assumes a model

$$u_t = B_1^* u_{t-1} + \dots + B_h^* u_{t-h} + \text{error}_t$$

and checks

$$H_0 : B_1^* = \dots = B_h^* = 0 \quad \text{vs.} \quad H_1 : B_1^* \neq 0 \text{ or } \dots \text{ or } B_h^* \neq 0.$$

The auxiliary model

$$\hat{u}_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + B_0 x_t + \dots + B_q x_{t-q} + CD_t + B_1^* \hat{u}_{t-1} + \dots + B_h^* \hat{u}_{t-h} + e_t \quad (4)$$

is considered. The model is estimated by the same method as the original model with \hat{u}_t , $t \leq 0$, replaced by zero. For example, for an unrestricted model multivariate LS estimation is used, whereas for a subset model EGLS is used.

Denoting the estimation residuals by \hat{e}_t ($t = 1, \dots, T$), the residual covariance matrix estimator obtained from the auxiliary models is

$$\tilde{\Sigma}_e = \frac{1}{T} \sum_{t=1}^T \hat{e}_t \hat{e}_t'$$

Moreover, reestimating the relevant auxiliary model without the lagged residuals \hat{u}_{t-i} , that is, imposing the restriction $B_1^* = \dots = B_h^* = 0$, and denoting the resulting residuals by \hat{e}_t^R , the corresponding covariance matrix estimator is

$$\tilde{\Sigma}_R = \frac{1}{T} \sum_{t=1}^T \hat{e}_t^R \hat{e}_t^{R'}$$

The LM statistic is

$$LM_h = T \left(K - \text{tr}(\tilde{\Sigma}_R^{-1} \tilde{\Sigma}_e) \right) \approx \chi^2(hK^2).$$

Edgerton and Shukur (1999) found that this test may be biased in small samples and therefore another statistic which may perform better is also given for full VAR models. It is of the form

$$LMF_h = \frac{1 - (1 - R_r^2)^{1/r}}{(1 - R_r^2)^{1/r}} \cdot \frac{Nr - q}{Km}$$

where

$$R_r^2 = 1 - \frac{|\tilde{\Sigma}_e|}{|\tilde{\Sigma}_R|}$$

with

$$r = \left(\frac{K^2 m^2 - 4}{K^2 + m^2 - 5} \right)^{1/2}, \quad q = \frac{1}{2} Km - 1, \quad N = T - K - m - \frac{1}{2}(K - m + 1),$$

n is the number of regressors in the original system and $m = Kh$ is the number of regressors added in the auxiliary system. The p -values of the statistic are based on an $F(hK^2, [Nr - q])$ distribution. Here $[Nr - q]$ denotes the largest integer less than or equal to $Nr - q$.

Tests for nonnormality

The idea underlying the nonnormality tests is to transform the residual vector such that its components are independent and then check the compatibility of the third and fourth moments with those of a normal distribution. In a first step, the residual covariance matrix is estimated as

$$\tilde{\Sigma}_u = T^{-1} \sum_{t=1}^T (\hat{u}_t - \bar{u})(\hat{u}_t - \bar{u})'$$

and the square root matrix $\tilde{\Sigma}_u^{1/2}$ is computed. The tests for nonnormality may be based on the skewness and kurtosis of the standardized residuals $\hat{u}_t^s = (\hat{u}_{1t}^s, \dots, \hat{u}_{Kt}^s)' = \tilde{\Sigma}_u^{-1/2}(\hat{u}_t - \bar{u})$:

$$\mathbf{b}_1 = (b_{11}, \dots, b_{1K})' \quad \text{with} \quad b_{1k} = T^{-1} \sum_{t=1}^T (\hat{u}_{kt}^s)^3$$

and

$$\mathbf{b}_2 = (b_{21}, \dots, b_{2K})' \quad \text{with} \quad b_{2k} = T^{-1} \sum_{t=1}^T (\hat{u}_{kt}^s)^4.$$

Defining

$$s_3^2 = T \mathbf{b}_1' \mathbf{b}_1 / 6$$

and

$$s_4^2 = T(\mathbf{b}_2 - \mathbf{3}_K)'(\mathbf{b}_2 - \mathbf{3}_K) / 24,$$

where $\mathbf{3}_K = (3, \dots, 3)'$ is a $(K \times 1)$ vector, a multivariate version of a Jarque-Bera statistic is

$$JB_K = s_3^2 + s_4^2.$$

The statistics s_3^2 and s_4^2 have $\chi^2(K)$ limiting distributions and JB_K has a $\chi^2(2K)$ asymptotic distribution if the normality null hypothesis holds. The latter statistic was proposed by Doornik and Hansen (1994).

An alternative way of computing standardized residuals was considered by Lütkepohl (1991, Chapter 4) who uses a Choleski decomposition of the residual covariance matrix. Let \tilde{P} be a lower triangular matrix with positive diagonal such that $\tilde{P}\tilde{P}' = \tilde{\Sigma}_u$. Then the residuals are standardized as $\hat{u}_t^s = \tilde{P}^{-1}(\hat{u}_t - \bar{u})$. Computing the third and fourth moments as in the foregoing as well as s_{3L}^2 and s_{4L}^2 corresponding to s_3^2 and s_4^2 , respectively, gives $JB_K^L = s_{3L}^2 + s_{4L}^2$ with asymptotic $\chi^2(2K)$ distribution under normality. Again, s_{3L}^2 and s_{4L}^2 have $\chi^2(K)$ limiting distributions.

Corresponding tests based on the univariate residual series are also given.

ARCH-LM test

A multivariate ARCH-LM test may be based on the multivariate regression model

$$\text{vech}(\hat{u}_t \hat{u}_t') = \beta_0 + B_1 \text{vech}(\hat{u}_{t-1} \hat{u}_{t-1}') + \dots + B_q \text{vech}(\hat{u}_{t-q} \hat{u}_{t-q}') + \text{error}_t, \quad (5)$$

where vech is the column stacking operator for symmetric matrices which stacks the columns from the main diagonal downwards, β_0 is $\frac{1}{2}K(K+1)$ -dimensional and the B_j are $(\frac{1}{2}K(K+1) \times \frac{1}{2}K(K+1))$ coefficient matrices ($j = 1, \dots, q$). The pair of hypotheses

$$H_0 : B_1 = \dots = B_q = 0 \quad \text{vs.} \quad H_1 : B_1 \neq 0 \text{ or } \dots \text{ or } B_q \neq 0,$$

is tested. It is checked by the multivariate LM statistic

$$VARCH_{LM}(q) = \frac{1}{2}TK(K+1)R_m^2,$$

where

$$R_m^2 = 1 - \frac{2}{K(K+1)}\text{tr}(\hat{\Omega}\hat{\Omega}_0^{-1}),$$

$\hat{\Omega}$ is the residual covariance matrix of the $\frac{1}{2}K(K+1)$ -dimensional regression model (5) and $\hat{\Omega}_0$ is the corresponding matrix with $q = 0$. The statistic is similar to the one described by Doornik and Hendry (1997, Sec. 10.9.2.4) and is based on a $\chi^2(qK^2(K+1)^2/4)$ distribution. Alternatively, an F version based on $VARCH_{LM}(q)/[qK^2(K+1)^2/4]$ may be used. In addition, analogous univariate ARCH tests can be applied to the individual residual series.

6.2 Implementation of Diagnostic Tests in JMulTi

The following tests are available:

Portmanteau Test is only available for models without exogenous variables, see Sec. 6.1

Tests for nonnormality multivariate and univariate versions are given, see Sec. 6.1

ARCH-LM multivariate and univariate versions can be selected, see Sec. 6.1

LM tests for autocorr. see Sec. 6.1

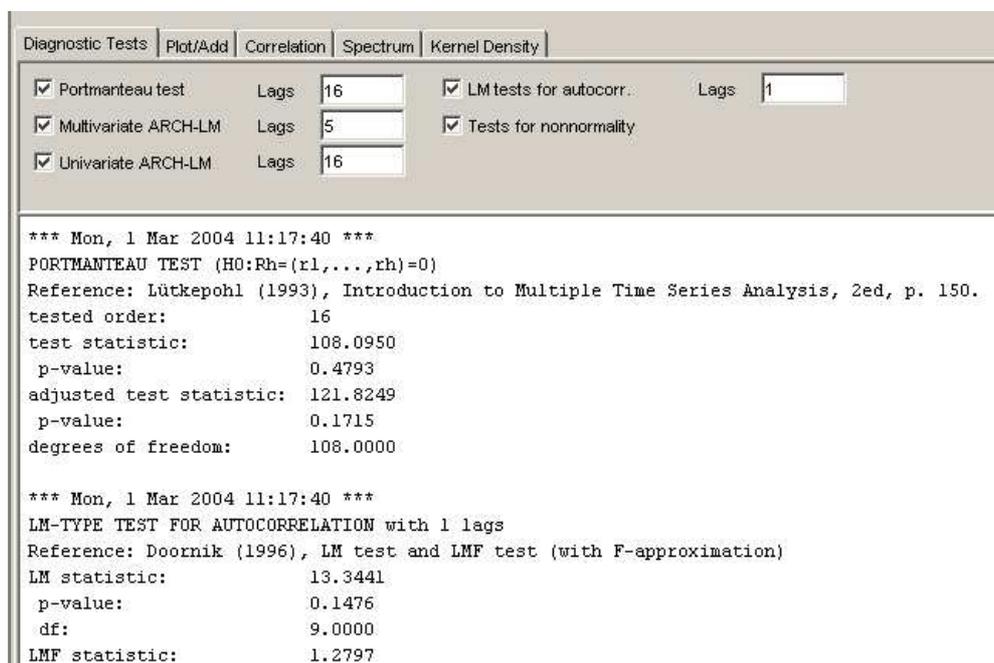


Figure 6: Diagnostic Tests in JMulTi

7 Correlation Analysis

7.1 Autocorrelation

In JMulTi residual autocorrelations (ACs) $\tilde{\rho}_{u,h} = \tilde{\gamma}_{u,h}/\tilde{\gamma}_{u,0}$ are obtained from

$$\tilde{\gamma}_{u,h} = \frac{1}{T} \sum_{t=h+1}^T (\hat{u}_t - \bar{\hat{u}})(\hat{u}_{t-h} - \bar{\hat{u}})$$

where $\bar{\hat{u}} = T^{-1} \sum_{t=1}^T \hat{u}_t$ is the sample mean.

The partial autocorrelation (PAC) between u_t and u_{t-h} is the conditional autocorrelation given $u_{t-1}, \dots, u_{t-h+1}$. The corresponding sample quantity \hat{a}_h is obtained as the OLS estimator of the coefficient α_h in an autoregressive model

$$\hat{u}_t = \nu + \alpha_1 \hat{u}_{t-1} + \dots + \alpha_h \hat{u}_{t-h} + error_t.$$

In JMulTi, OLS estimates are obtained for each h with sample size $T - h$. The approximate 95% confidence bounds defined by $\pm 2/\sqrt{T}$ are used. ACs and PACs are computed for the single residual series.

7.2 Crosscorrelation

To get an overall picture of the correlation structure between the different residual series, a crosscorrelation plot may be used. In JMulTi we follow the definition of the exact asymptotic confidence intervals as described in Lütkepohl (1991), Sec. 4.4.2, for stable, unrestricted VARs and Sec. 5.2.9 for stable VARs with parameter constraints. In case there are exogenous variables in the model, only the standard $\pm 2/\sqrt{T}$ confidence bounds are available.

7.3 Plotting Autocorrelations and Crosscorrelations in JMulTi

The available residuals are shown in the two lists from which the ones have to be selected for which autocorrelations or crosscorrelations are desired. The selection is done as usual by highlighting the residual series.

plotting AC/PAC select the desired series from the left list and press the *Autocorrelation* button

plotting crosscorrelations select the desired combination of series in the two lists and press the *Crosscorrelation* button

Number of lags the autocorrelation functions are computed up to the specified lag order

Squared residuals the residuals are squared before the autocorrelations are computed, exact asymptotic confidence intervals are not available then

Exact CI for crosscorr. in case there are no exogenous variables, the exact asymptotic 95% confidence intervals are estimated, otherwise the standard $\pm 2/\sqrt{T}$ confidence bounds are used

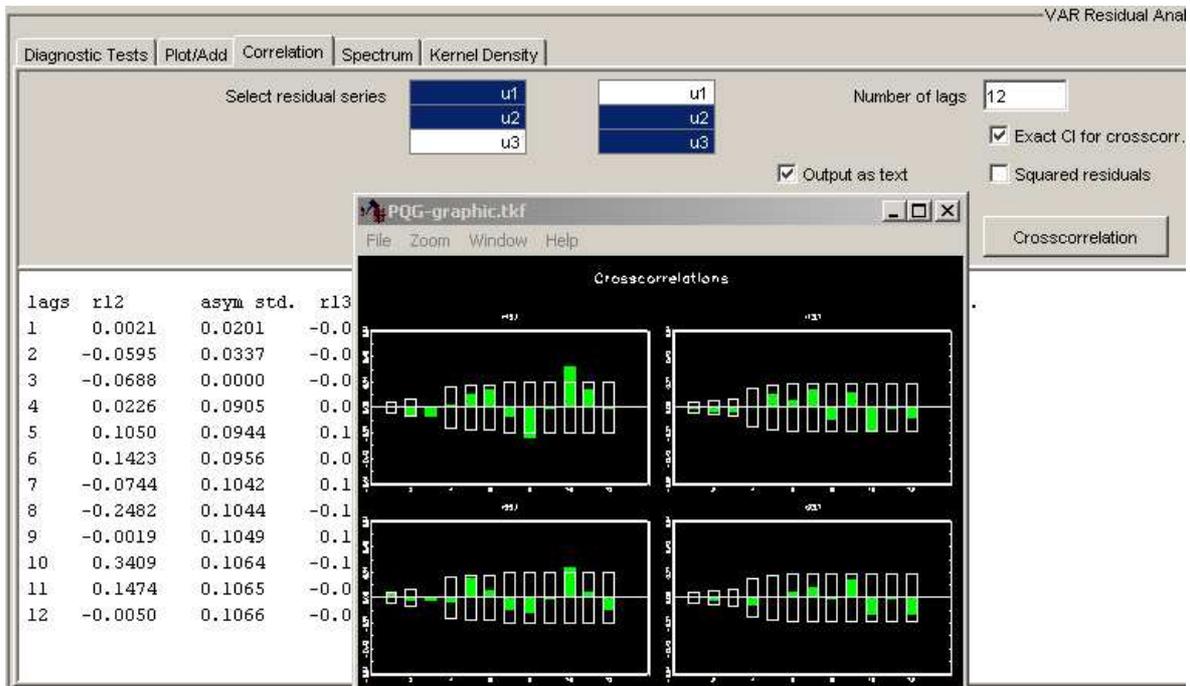


Figure 7: Correlation Analysis in JMulTi

8 Stability Analysis

Parameter constancy throughout the sample period is a key assumptions in econometric models. JMulTi offers several options to check its validity: recursive residuals and parameter estimates, Chow tests and CUSUM tests.

8.1 Recursive parameter estimates

Recursive parameter estimates are a simple descriptive tool for assessing parameter stability. They are obtained by simply estimating the model using only data for $t = 1, \dots, \tau$ and letting τ vary from some small value to T , the end of the original sample. Thereby sequences of coefficient estimates and estimates of the covariance matrix of the asymptotic distributions are obtained. Here the same estimation method is used which is also used for the full sample estimation. For example, for a subset VAR model, feasible GLS is used. The series of estimates together with two-standard error bands are then plotted and can convey useful information on the relative importance of new observations that are added to the sample.

Implementation in JMulTi

To get to the recursive parameter estimation panel you need to select *Model Checking* \rightarrow *Stability Analysis* \rightarrow *Recursive Coefficients*. The full model as specified in the selection panel is displayed for the user to choose the coefficients for which the graphs of recursive estimates are to be produced. Several choices have to be made before displaying the graphs.

Start date marks the beginning of the recursive estimation

Each coeff in separate window displays each coefficient in a separate graph.

Group different lags in separate windows displays graphs with each containing all the coefficients of a parameter matrix as shown in the selection panel.

8.2 Recursive residuals

The so-called recursive residuals are standardized 1-step forecast errors from a model estimated on the basis of data up to period $\tau - 1$. They are computed for the individual equations of a vector model separately. For a single equation model $y_t = x_t' \beta + u_t$ ($t = 1, \dots, T$) with x_t ($M \times 1$), they are obtained as follows. Denoting by $\hat{\beta}_{(\tau)}$ the OLS estimator based on the first τ observations only, that is,

$$\hat{\beta}_{(\tau)} = \left(\sum_{t=1}^{\tau} x_t x_t' \right)^{-1} \sum_{t=1}^{\tau} x_t y_t, \quad \tau \geq M,$$

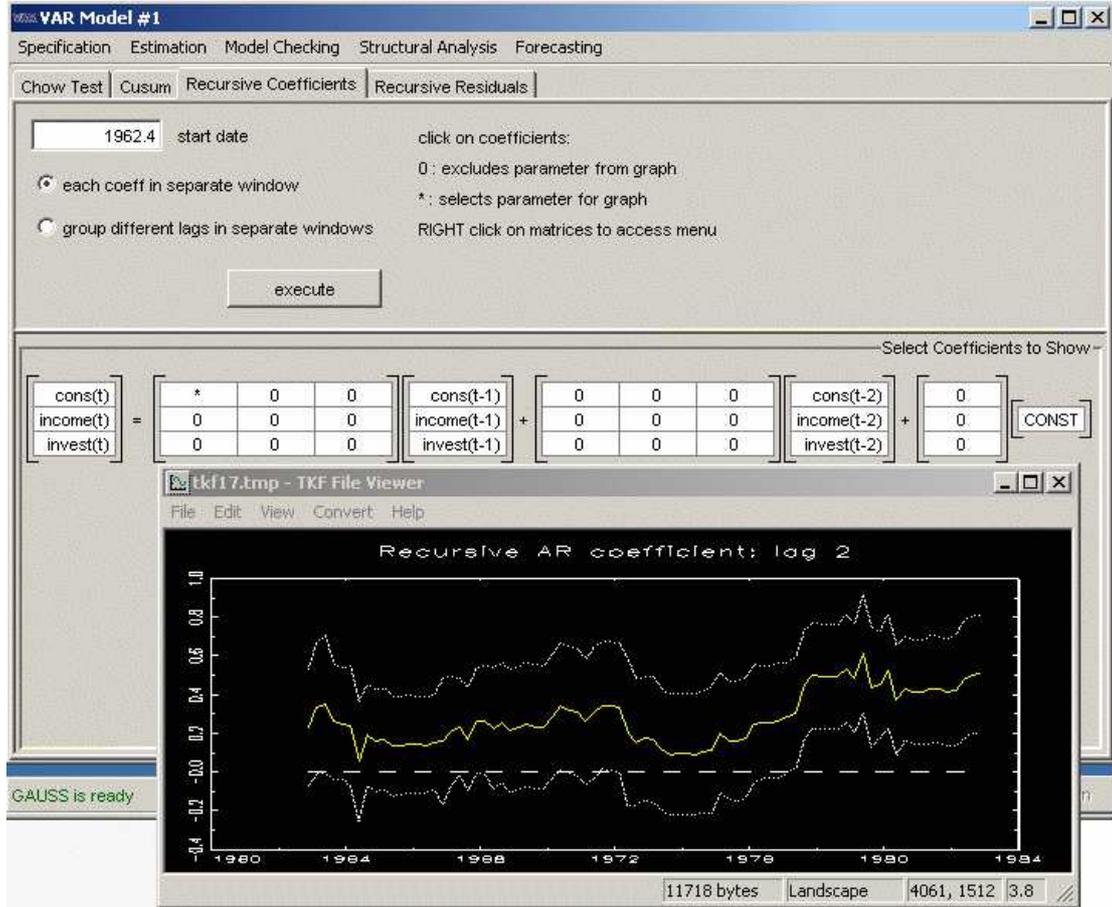


Figure 8: Recursive parameter estimates.

the recursive residuals are defined as

$$\hat{u}_\tau^{(r)} = \frac{y_\tau - x'_\tau \hat{\beta}_{(\tau-1)}}{\left(1 + x'_\tau \left(\sum_{t=1}^{\tau-1} x_t x'_t\right)^{-1} x_\tau\right)^{1/2}}, \quad \tau = M + 1, \dots, T.$$

If x_t consists of fixed, nonstochastic regressors, the forecast error $y_\tau - x'_\tau \hat{\beta}_{(\tau-1)}$ is known to have mean zero and variance $\sigma_u^2 \left(1 + x'_\tau \left(\sum_{t=1}^{\tau-1} x_t x'_t\right)^{-1} x_\tau\right)$. Hence, the recursive residuals have constant variance σ_u^2 . Therefore, even if some of the regressors are stochastic, the recursive residuals are often plotted with "confidence intervals" $\pm c_{1-\gamma/2} \hat{\sigma}_u$ bands, where $c_{1-\gamma/2}$ is the relevant quantile from a normal distribution table and

$$\hat{\sigma}_u^2 = (T - M)^{-1} \sum_{t=1}^T \hat{u}_t^2$$

is the usual residual variance estimator based on the full sample. In other words, $\hat{u}_t = y_t - x'_t \hat{\beta}_{(T)}$. The recursive residuals exist only if the inverse of $\sum_{t=1}^{\tau} x_t x'_t$ exists for all $\tau = M + 1, \dots, T$. Thus they may not be available in the presence of dummy variables.

Implementation in JMulTi

To get to the recursive residuals panel you need to select *Model Checking* → *Stability Analysis* → *Recursive Residuals*. Two choices can be made before displaying the graphs.

Standardize residuals are the recursive residuals divided by the estimated standard deviation $\hat{\sigma}_u$ based on the original OLS residuals. If checked, JMulTi displays the graphs of the standardized residuals together with horizontal lines ± 1 and ± 2 to facilitate the visual inspection of the graphs. If this option is not selected, JMulTi displays the graphs of recursive residuals together with "confidence intervals"

Coverage probability of CIs allows you to choose the confidence level 90%, 95% or 99% for the "confidence intervals" around the recursive residuals

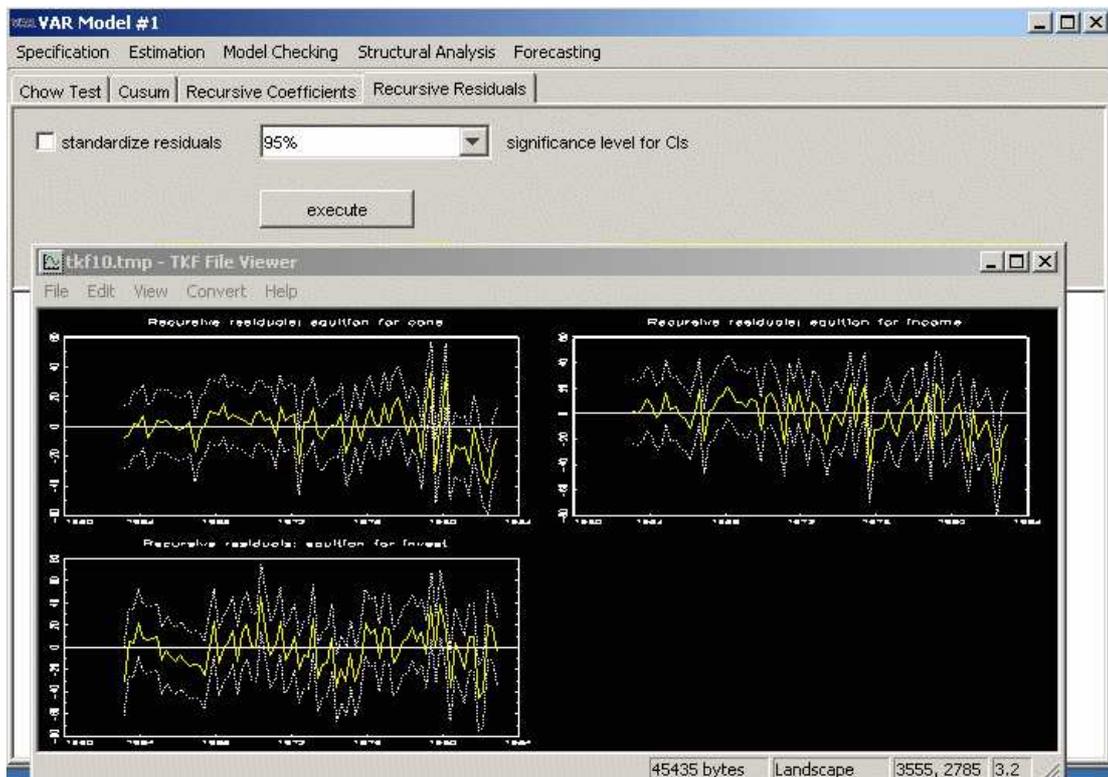


Figure 9: Recursive residuals.

8.3 CUSUM tests

The cumulative sum of recursive residuals up to period τ is

$$CUSUM_{\tau} = \sum_{t=M+1}^{\tau} \hat{u}_t^{(r)} / \hat{\sigma}_u.$$

It is plotted for $\tau = M+1, \dots, T$ together with the lines $\pm c_\gamma[\sqrt{T-M} + 2(\tau-M)/\sqrt{T-M}]$, where c_γ depends on the desired significance level of the resulting test. Brown, Durbin and Evans (1975) show that if the CUSUMs wander beyond these lines, this is evidence against structural stability of the underlying model.

Sometimes the CUSUM-of-squares plot based on

$$CUSUM - SQ_\tau = \frac{\sum_{t=M+1}^{\tau} (\hat{u}_t^{(r)})^2}{\sum_{t=M+1}^T (\hat{u}_t^{(r)})^2}$$

results in a more powerful test. If the $CUSUM - SQ_\tau$ cross the lines given by $\pm c + (\tau - M)/(T - M)$, structural instability is indicated. The constant c depends on the desired significance level, the sample size T and the number of regressors in the model. It was also proposed by Brown et al. (1975).

In JMulTi CUSUM and CUSUM-SQ tests are provided for each equation separately.

Implementation in JMulTi

Handling of dummy variables Modifications are necessary if there are dummy variables which are constant for the first τ observations. In such a case the formula for the computation of the recursive residuals does not apply anymore. In this case, following Brown et al. (1975), the dummy is dropped from the estimation of the recursive residuals for the initial part of the sample (now starting estimation not from observation $M+1$ but from M) and bringing it into the regression when the parameter associated with the dummy regressor is estimable. Note that there will be no residual for the period where the dummy is added to the regressors because there is no meaningful forecast for the related period but the overall number of residuals will still be the same because there is an additional residual at the beginning.

Significance level of the test allows you to choose the significance level 10%, 5% or 1% for the CUSUM and CUSUM-of-squares tests

There is also a possibility to choose between the CUSUM and CUSUM-of-squares tests.

8.4 Chow tests

In JMulTi, break-point (BP), sample-split (SS) and Chow forecast (CF) tests are available. They are applied to the full system rather than the individual equations. The BP Chow test for checking for a structural break in period T_B proceeds as follows. The model under consideration is estimated from the full sample of T observations and from the first T_1 and the last T_2 observations, where $T_1 < T_B$ and $T_2 \leq T - T_B$. Denoting the resulting

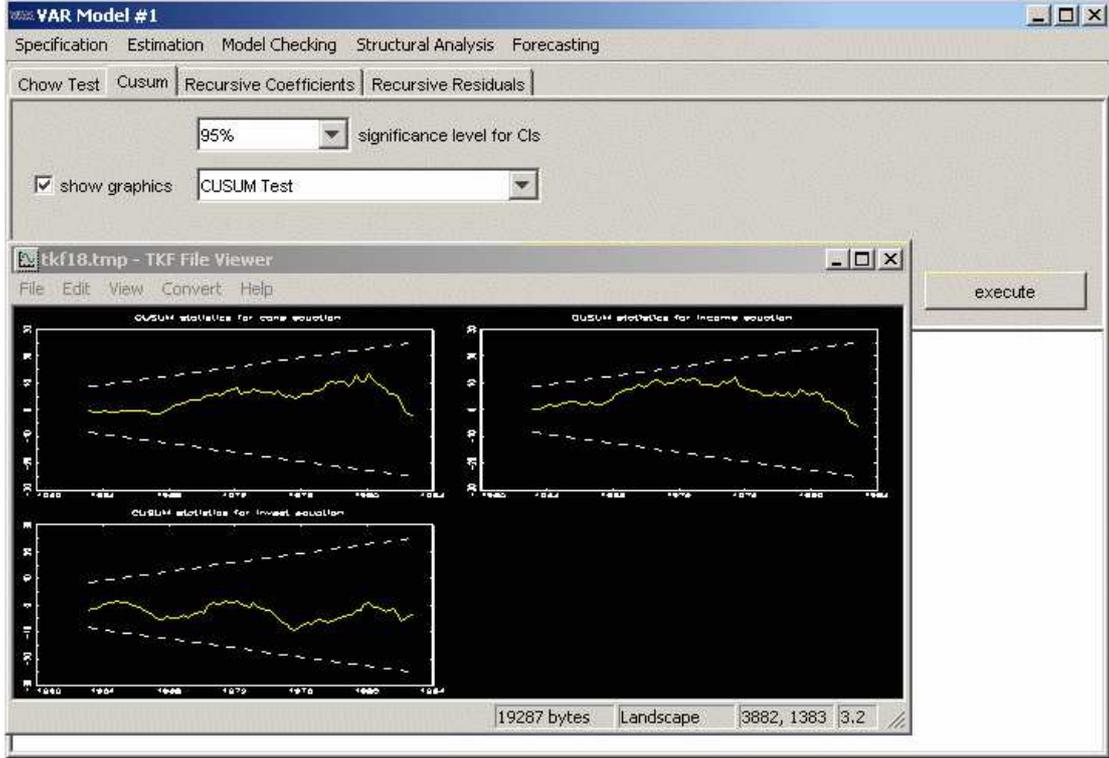


Figure 10: CUSUM and CUSUM-of-squares tests.

residuals by \hat{u}_t , $\hat{u}_t^{(1)}$ and $\hat{u}_t^{(2)}$, respectively, and using the notation $\tilde{\Sigma}_u = T^{-1} \sum_{t=1}^T \hat{u}_t \hat{u}_t'$, $\tilde{\Sigma}_{1,2} = (T_1 + T_2)^{-1} (\sum_{t=1}^{T_1} \hat{u}_t \hat{u}_t' + \sum_{t=T-T_2+1}^T \hat{u}_t \hat{u}_t')$, $\tilde{\Sigma}_{(1)} = T_1^{-1} \sum_{t=1}^{T_1} \hat{u}_t^{(1)} \hat{u}_t^{(1)'}$ and $\tilde{\Sigma}_{(2)} = T_2^{-1} \sum_{t=T-T_2+1}^T \hat{u}_t^{(2)} \hat{u}_t^{(2)'}$, the BP test statistic is

$$\lambda_{BP} = (T_1 + T_2) \log \det \tilde{\Sigma}_{1,2} - T_1 \log \det \tilde{\Sigma}_{(1)} - T_2 \log \det \tilde{\Sigma}_{(2)} \approx \chi^2(k).$$

Here k is the difference between the sum of the number of parameters estimated in the first and last subperiods and the number of parameters in the full sample model. Note that also the potentially different parameters in the white noise covariance matrix are counted. The null hypothesis of constant parameters is rejected if λ_{BP} is large.

The SS statistic is derived under the assumption that the residual covariance matrix Σ_u is constant and checks against the alternative that the VAR coefficients may vary. It has the form

$$\lambda_{SS} = (T_1 + T_2) [\log \det \tilde{\Sigma}_{1,2} - \log \det \{(T_1 + T_2)^{-1} (T_1 \tilde{\Sigma}_{(1)} + T_2 \tilde{\Sigma}_{(2)})\}] \approx \chi^2(k^-).$$

Here k^- is the difference between the sum of the number of coefficients estimated in the first and last subperiods and the number of coefficients in the full sample model, not counting the parameters in the white noise covariance matrix.

The CF statistic is

$$\lambda_{CF} = \frac{1 - (1 - R_r^2)^{1/s}}{(1 - R_r^2)^{1/s}} \cdot \frac{Ns - q}{Kk^*} \approx F(Kk^*, [Ns - q]),$$

where

$$s = \left(\frac{K^2 k^{*2} - 4}{K^2 + k^{*2} - 5} \right)^{1/2}, \quad q = \frac{Kk^*}{2} + 1, \quad N = T - k_1 - k^* - (K - k^* + 1)/2.$$

Here k_1 is the number of regressors in each equation of the time invariant model, k^* the number of forecast periods considered by the test ($k^* = T - T_1$), and

$$R_r^2 = 1 - \left(\frac{T_1}{T} \right)^K |\tilde{\Sigma}_{(1)}| (|\tilde{\Sigma}_u|)^{-1}.$$

This test is only available for full models and not for subset VARs. For the second degrees of freedom, $Ns - q$, of the approximating F distribution, the integer part, $[Ns - q]$, is used whenever $Ns - q$ is not an integer. The CF test tests against the alternative that all coefficients including the residual covariance matrix may vary. It also rejects the null hypothesis of constant parameters for large values of the test statistic.

Because the actual small sample distributions of the test statistics under H_0 may be quite different from the asymptotic χ^2 - or F -distributions (see Candelon and Lütkepohl (2000)), JMulTi offers bootstrap p -values. They are computed as follows. From the estimation residuals \hat{u}_t , centered residuals $\hat{u}_1 - \bar{\hat{u}}, \dots, \hat{u}_T - \bar{\hat{u}}$ are computed. Bootstrap residuals u_1^*, \dots, u_T^* are generated by randomly drawing with replacement from the centered residuals. Based on these quantities, bootstrap time series are calculated recursively starting from given pre-sample values y_{-p+1}, \dots, y_0 . Then the model is reestimated with and without allowing for a break and bootstrap versions of the statistics of interest, say λ_{BP}^* , λ_{SS}^* and λ_{CF}^* are computed. The p -values of the tests are estimated as the proportions of values of the bootstrap statistics exceeding the corresponding test statistic based on the original sample.

Dealing with dummies In practice, one often has regression models with a constant, where in addition several of the remaining regressor variables are dummies which might also be constant either for the first or for the second part of the sample created by splitting the sample at T_B . In order to avoid perfect collinearity between the regressors, JMulTi deletes dummies automatically from the estimation of either the first or the second part of the sample where appropriate with corresponding adjustment of the degrees of freedom if necessary.

Specification in JMulTi

Chow tests can be performed for individual time periods or for a range of time points. In the latter case the results are given in tables as well as graphs.

Input

Break date allows you to specify a break period for which Chow tests are to be performed

Search over datapoints selecting this option allows you to perform Chow tests not only for a single break date but over a range of the time points as specified in the adjacent menu entry

Test range allows you to choose the starting and ending dates for the *Search over datapoints* procedure

Graph of bootstrapped p-values checking this option produces graphs of p -values of the BP, SS and CF tests calculated for each break date in the specified range

Number of bootstrap replications specifies the number of replications used to construct the empirical distribution function of the statistics. It has a substantial impact on the required computation time. For reliable results it may be necessary to select a few thousand replications although this may result in substantial waiting times

Output

break point Chow test (chow-bp) value of BP test statistic

boot p-val shows the bootstrapped p -value of the corresponding statistic

chi² p-val shows the p -values of the approximating χ^2 distribution corresponding to the BP statistic

df degrees of freedom of the corresponding χ^2 distribution

sample split Chow test (chow-ss) value of SS test statistic

boot p-val shows the bootstrapped p -value of the corresponding statistic

chi² p-val shows the p -values of the approximating χ^2 distribution corresponding to the SS statistic

df degrees of freedom of the corresponding χ^2 distribution

Chow forecast test (chow-fc) values of CF test statistics

F p-val shows the p -values of the approximating F distribution corresponding to the CF statistic, only available if a full VAR model has been fitted

df1 numerator degrees of freedom of the corresponding F distribution

df2 denominator degrees of freedom of the corresponding F distribution, integer part of $Ns - q$

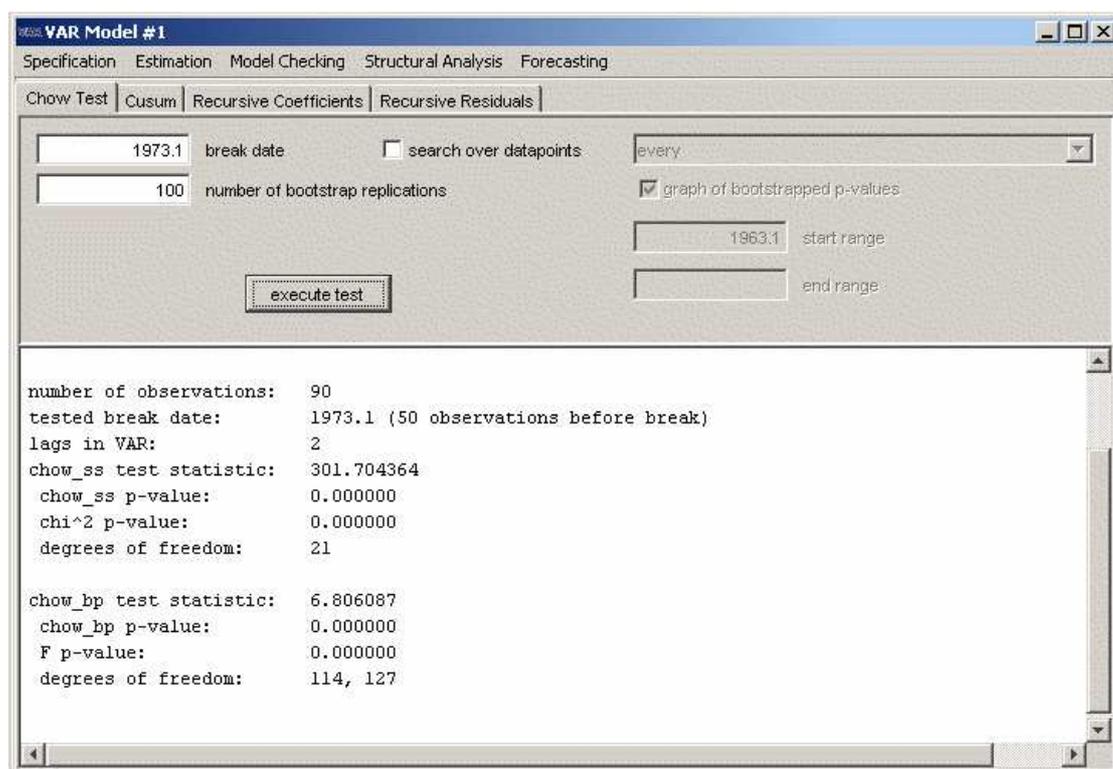


Figure 11: Chow tests for a single break point.

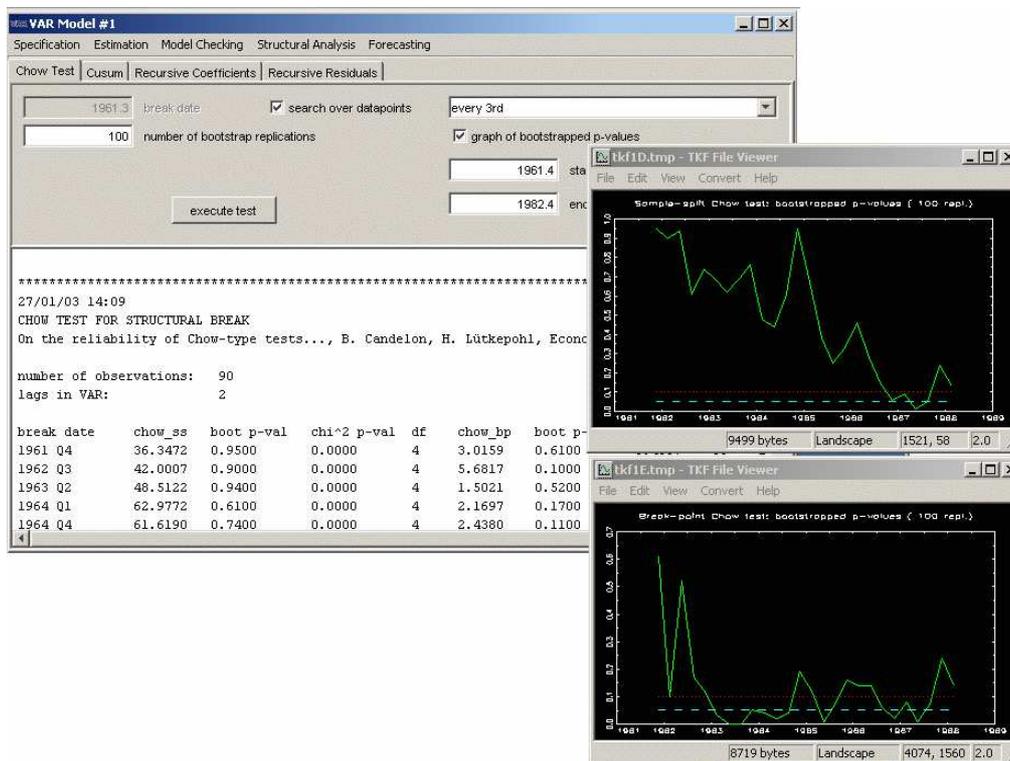


Figure 12: Chow tests for a range of possible break points.

9 Causality Analysis

9.1 Background

Two types of causality tests are implemented in `JMulTi`, tests for Granger-causality and tests for instantaneous causality. For both types of tests the vector of endogenous variables is divided in two subvectors, y_{1t} and y_{2t} , with dimensions K_1 and K_2 , respectively, so that $K = K_1 + K_2$. The subvector y_{1t} is said to be Granger-causal for y_{2t} if it contains useful information for predicting the latter set of variables. For testing this property, a model of the form

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \sum_{i=1}^p \begin{bmatrix} \alpha_{11,i} & \alpha_{12,i} \\ \alpha_{21,i} & \alpha_{22,i} \end{bmatrix} \begin{bmatrix} y_{1,t-i} \\ y_{2,t-i} \end{bmatrix} + CD_t + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix},$$

is considered. In this model setup, y_{1t} is not Granger-causal for y_{2t} if and only if

$$\alpha_{21,i} = 0, \quad i = 1, 2, \dots, p.$$

Therefore this null hypothesis is tested against the alternative that at least one of the $\alpha_{21,i}$ is nonzero. A Wald test statistic, divided by the number of restrictions pK_1K_2 , is used in conjunction with an $F(pK_1K_2, KT - n^*)$ distribution for testing the restrictions. Here n^* is the total number of parameters in the system (see Lütkepohl (1991)), including the parameters of the deterministic term. Of course, the role of y_{1t} and y_{2t} can be reversed to test Granger-causality from y_{2t} to y_{1t} .

The test is problematic if some of the variables are nonstationary (integrated). In that case the usual asymptotic distribution of the test statistic may not be valid under the null hypothesis. Therefore, the test should be performed in the VEC framework if there are integrated variables in the system of interest.

Instantaneous causality is characterized by nonzero correlation of u_{1t} and u_{2t} . Thus the null hypothesis

$$H_0 : \mathbf{E}(u_{1t}u_{2t}') = 0$$

is tested against the alternative of nonzero covariance between the two error vectors in testing for instantaneous causality. The Wald test described in Lütkepohl (1991, Sec. 3.6.3) is reported in `JMulTi`.

If there are exogenous variables in the model, the analysis is carried out conditionally on these variables. In other words, a model

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \sum_{i=1}^p \begin{bmatrix} \alpha_{11,i} & \alpha_{12,i} \\ \alpha_{21,i} & \alpha_{22,i} \end{bmatrix} \begin{bmatrix} y_{1,t-i} \\ y_{2,t-i} \end{bmatrix} + B_0x_t + \dots + B_qx_{t-q} + CD_t + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

is considered and the tests are carried out on the α coefficients and the covariance of u_{1t} and u_{2t} in this model.

9.2 Causality Analysis in JMulTi

After the model is specified with a VAR order of at least one, see Sec. 3, the causality analysis can be accessed by clicking on the menu *Structural Analysis*→*Causality Tests*. The endogenous variables are shown in the list. There you can select from 1 up to $K - 1$ variables. The respective H_0 hypothesis then appears on the panel. The tests are always performed on the unrestricted VAR part of the model even if a subset model is currently under consideration. Restrictions for exogenous variables and deterministic terms are maintained, however. In other words, if zero restrictions have been placed on the coefficient matrices of the exogenous variables and/or the deterministic terms, these restrictions will be imposed in the causality tests.

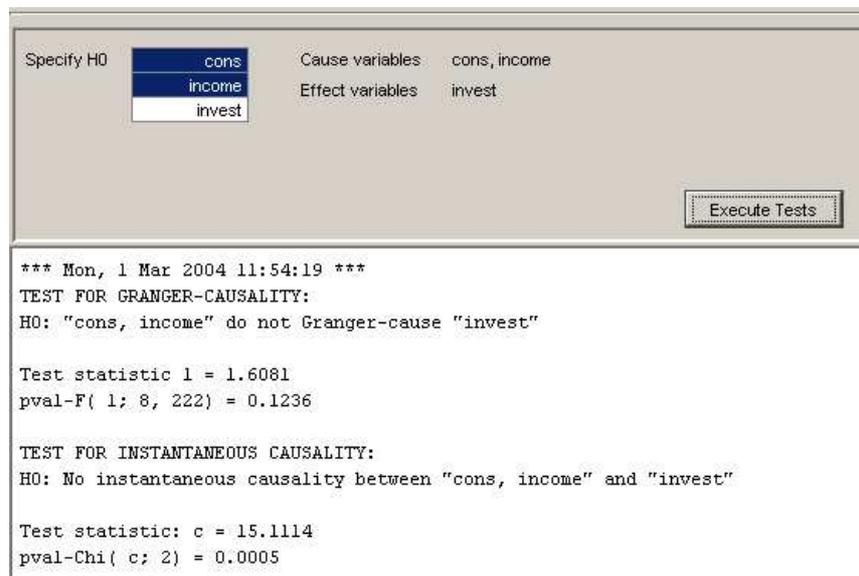


Figure 13: Causality Analysis in JMulTi

10 Impulse Response Analysis

10.1 Background

Impulse response analysis can be used to analyze the dynamic interactions between the endogenous variables of a VAR(p) process. In this analysis the exogenous and deterministic variables are treated as fixed and may therefore be dropped from the system. In other words, the part of the conditional mean of the endogenous variables attributable to these variables is eliminated. The adjusted endogenous variables are now denoted by y_t . If the process y_t is stationary ($I(0)$), it has a Wold moving average (MA) representation

$$y_t = \Phi_0 u_t + \Phi_1 u_{t-1} + \Phi_2 u_{t-2} + \cdots, \quad (6)$$

where $\Phi_0 = I_K$ and the Φ_s can be computed recursively as

$$\Phi_s = \sum_{j=1}^s \Phi_{s-j} A_j, \quad s = 1, 2, \dots,$$

with $\Phi_0 = I_K$ and $A_j = 0$ for $j > p$. The coefficients of this representation may be interpreted as reflecting the responses to impulses hitting the system. The (i, j) th elements of the matrices Φ_s , regarded as a function of s , trace out the expected response of $y_{i,t+s}$ to a unit change in y_{jt} holding constant all past values of y_t . The elements of Φ_s represent the impulse responses of the components of y_t with respect to the u_t innovations. These impulse responses are sometimes called *forecast error impulse responses* because the u_t are the 1-step ahead forecast errors (see Lütkepohl (1991) for further discussion).

Although the Wold representation does not exist for nonstationary cointegrated processes, the Φ_s impulse response matrices can be computed in the same way for nonstationary processes. Thus the forecast error impulse responses are available even if some variables are not $I(0)$. In contrast to the stationary case, impulses hitting a nonstationary system may have permanent effects, however.

Because the underlying shocks are not likely to occur in isolation if the components of u_t are not instantaneously uncorrelated, that is, if Σ_u is not diagonal, in many applications the innovations of the VAR are orthogonalized using a Cholesky decomposition of the covariance matrix Σ_u . Denoting by P a lower triangular matrix such that $\Sigma_u = PP'$, the orthogonalized shocks are given by $\varepsilon_t = P^{-1}u_t$. Hence, in the stationary case we get,

$$y_t = \Psi_0 \varepsilon_t + \Psi_1 \varepsilon_{t-1} + \cdots, \quad (7)$$

where $\Psi_i = \Phi_i P$ ($i = 0, 1, 2, \dots$). Here $\Psi_0 = P$ is lower triangular so that an ε shock in the first variable may have an instantaneous effect on all the variables, whereas a shock in the second variable cannot have an instantaneous impact on y_{1t} but only on the other variables and so on.

Notice that if a different ordering of the variables in the vector y_t is chosen this may produce different impulse responses. Hence, the effects of a shock may depend on the way the variables are arranged in the vector y_t . In view of this nonuniqueness of the impulse responses *structural VAR* analysis has been developed (see Sec. 13).

10.2 Impulse Response Analysis in JMulTi

To access the Impulse Response Panel, select *Structural Analysis*→*Impulse Response Analysis* from the menu. It is possible to create orthogonalized impulse responses based on an innovation of size one standard deviation in the transformed model as well as forecast error variance impulse responses based on a unit innovation in the original model.

First you have to select the impulse and response variables as well as the desired confidence interval from the tables. Then you can generate text or graphical output. The confidence intervals are only available if they have been generated before, see Sec. 11.

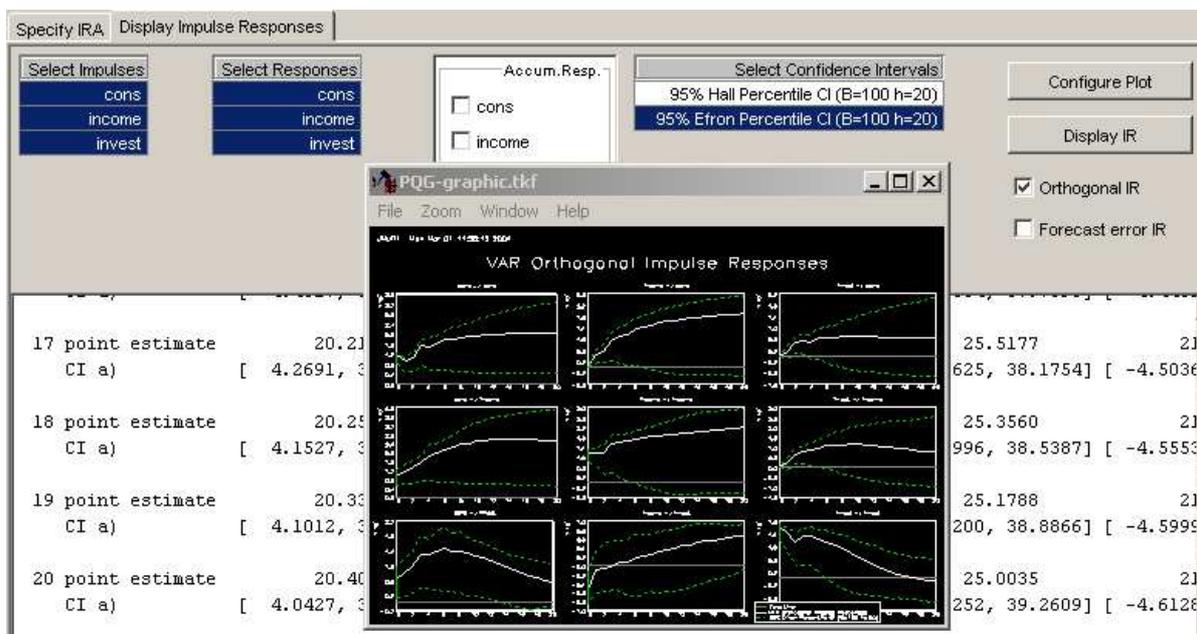


Figure 14: Impulse Response Analysis in JMulTi

11 Bootstrapping Impulse Responses

11.1 Background

In practice, the impulse responses are computed from the estimated VAR coefficients and bootstrap methods are available in JMulTi to construct confidence intervals (CIs) which reflect the estimation uncertainty. Alternative bootstrap approaches are implemented. They proceed as follows:

First the model of interest is estimated. Denoting the estimation residuals by \hat{u}_t the centered residuals $\hat{u}_1 - \bar{\hat{u}}, \dots, \hat{u}_T - \bar{\hat{u}}$ are obtained. Then bootstrap residuals u_1^*, \dots, u_T^* are generated by randomly drawing with replacement from the centered residuals. These quantities are used to compute bootstrap time series recursively starting from given presample values y_{-p+1}, \dots, y_0 and fixing the exogenous and deterministic terms. The model of interest is then reestimated and bootstrap versions of the quantities of interest are computed. Repeating these steps a large number of times, bootstrap distributions of the quantities of interest are obtained.

In the following the symbols ϕ , $\hat{\phi}$ and $\hat{\phi}^*$ denote some general impulse response coefficient, its estimator implied by the estimators of the model coefficients and the corresponding bootstrap estimator, respectively. The following bootstrap CIs are implemented in JMulTi :

- *Standard percentile interval*

It is determined as

$$CI_S = [s_{\gamma/2}^*, s_{(1-\gamma/2)}^*],$$

where $s_{\gamma/2}^*$ and $s_{(1-\gamma/2)}^*$ are the $\gamma/2$ - and $(1 - \gamma/2)$ -quantiles, respectively, of the bootstrap distribution of $\hat{\phi}^*$. The interval CI_S is the percentile confidence interval described, e.g., by Efron and Tibshirani (1993).

- *Hall's percentile interval*

It is determined as

$$CI_H = [\hat{\phi} - t_{(1-\gamma/2)}^*, \hat{\phi} - t_{\gamma/2}^*],$$

where $t_{\gamma/2}^*$ and $t_{(1-\gamma/2)}^*$ are the $\gamma/2$ - and $(1 - \gamma/2)$ -quantiles, respectively, of the distribution of $\hat{\phi}^* - \hat{\phi}$ (see Hall (1992)).

- *Hall's studentized interval*

It is determined by using bootstrap quantiles $t_{\gamma/2}^{**}$ and $t_{1-\gamma/2}^{**}$ from the distribution of $(\hat{\phi}^* - \hat{\phi})/(\widehat{\text{var}}(\hat{\phi}^*))^{1/2}$,

$$CI_{SH} = [\hat{\phi} - t_{(1-\gamma/2)}^{**}(\widehat{\text{var}}(\hat{\phi}))^{1/2}, \hat{\phi} - t_{\gamma/2}^{**}(\widehat{\text{var}}(\hat{\phi}))^{1/2}].$$

In this approach the variances are estimated by a bootstrap within each bootstrap replication. Therefore it is rather demanding in terms of computing time.

Unfortunately, the bootstrap does not always result in CIs with the desired coverage even asymptotically. For a critical discussion see Benkwitz, Lütkepohl and Neumann (2000).

11.2 Bootstrapping Confidence Intervals in JMulTi

Because bootstrapping confidence intervals for impulse responses may be time consuming, it is in a separate panel, where the relevant parameters can be adjusted. The generated CIs are saved until you either specify a different model or you increase the number of lags for the impulse response analysis.

Use this seed you may use a distinct seed value to initialize the random number generator with a certain value, if disabled, the random number generator is initialized each time it is invoked with a different value dependent on the system time

Number of bootstrap replications this value affects computing time, for reliable CIs the number still has to be large, say a few thousand

Number of periods the number of periods for which the CIs as well as the impulse responses are computed, increasing this value deletes already computed CIs

Select the type of CI you want to generate from the available combo box. There are two panels available. For *Efron & Hall Bootstrap Percentile CI*, the *Standard percentile interval*, see Sec. 11.1, as well as *Hall's percentile interval*, see Sec. 11.1, are generated. The other option is to generate *Studentized Hall Bootstrap CI*, see Sec. 11.1, with the additional computation of the variance in each step.

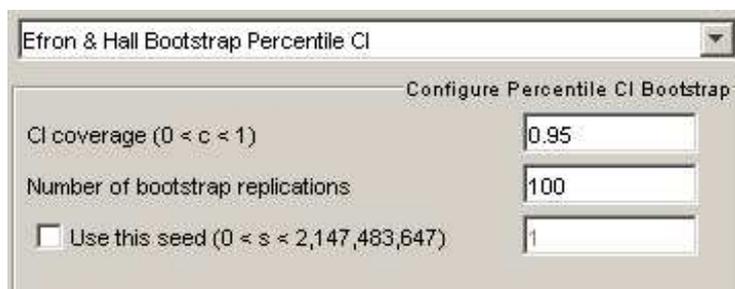


Figure 15: Bootstrapping Efron & Hall Confidence Intervals in JMulTi

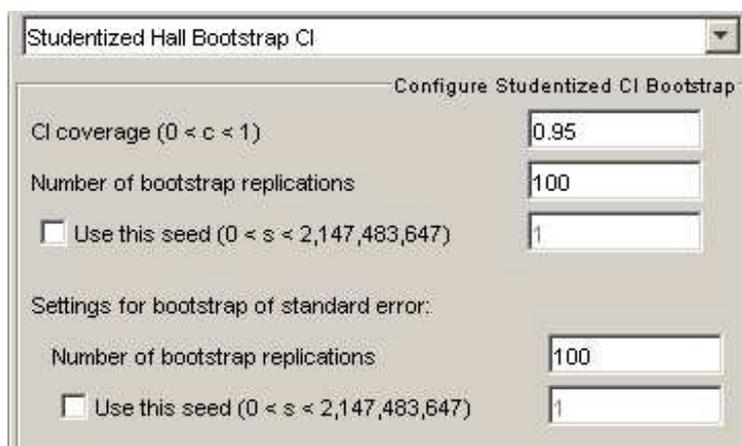


Figure 16: Bootstrapping Hall's Studentized Confidence Intervals in JMulti

12 Forecast Error Variance Decomposition

12.1 Background

Forecast error variance decompositions (FEVDs) are popular tools for interpreting VAR models. Denoting the ij th element of the orthogonalized impulse response coefficient matrix Ψ_n by $\psi_{ij,n}$, the variance of the forecast error

$$y_{k,T+h} - y_{k,T+h|T}$$

is

$$\sigma_k^2(h) = \sum_{n=0}^{h-1} (\psi_{k1,n}^2 + \cdots + \psi_{kK,n}^2) = \sum_{j=1}^K (\psi_{kj,0}^2 + \cdots + \psi_{kj,h-1}^2).$$

The term $(\psi_{kj,0}^2 + \cdots + \psi_{kj,h-1}^2)$ is interpreted as the contribution of variable j to the h -step forecast error variance of variable k . Dividing the above terms by $\sigma_k^2(h)$ gives the percentage contribution of variable j to the h -step forecast error variance of variable k ,

$$\omega_{kj}(h) = (\psi_{kj,0}^2 + \cdots + \psi_{kj,h-1}^2) / \sigma_k^2(h)$$

(see Lütkepohl (1991)). In JMulTi these quantities, computed from estimated parameters, are reported for various forecast horizons.

12.2 Implementation in JMulTi

To access the FEVD panel after a model was estimated, select *Structural Analysis*→*FEV Decomposition* from the menu. You may then select the variables which should be decomposed from the table. A plot is created, which presents the contributions of each variable to the FEV of the selected variable in a bar diagram. Text output is also provided. It can be saved by RIGHT clicking over the text area.

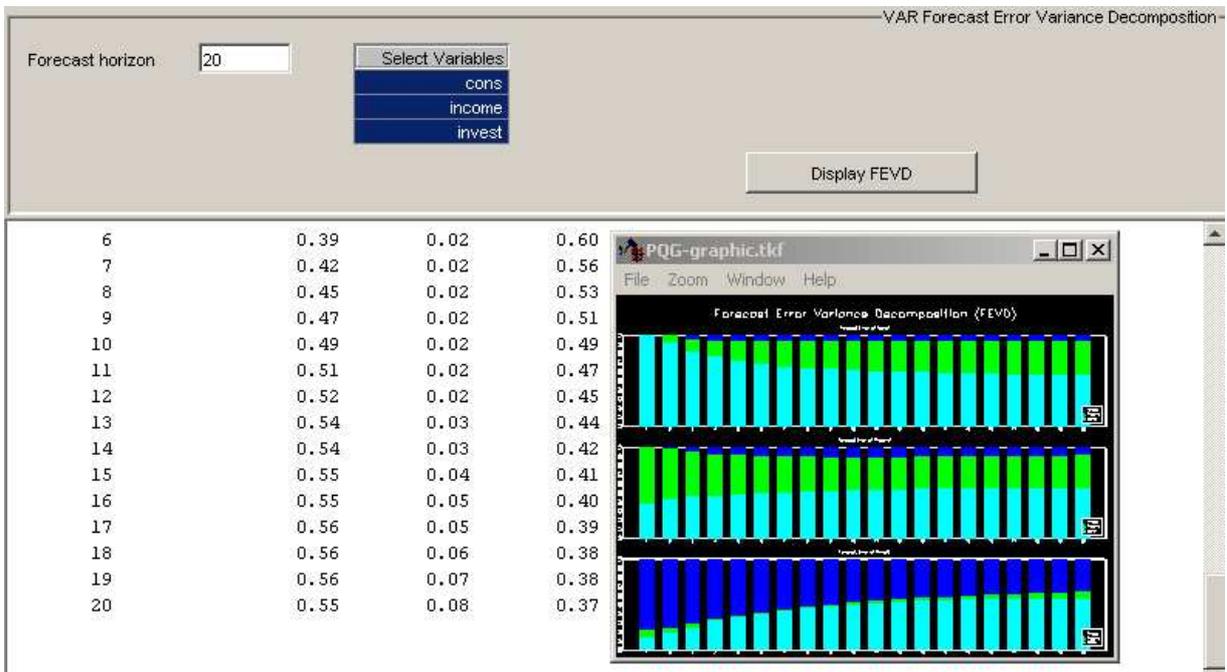


Figure 17: FEVD in JMulti

13 SVAR Analysis

The SVAR (structural vector autoregressive) model can be used to identify the shocks to be traced in an impulse response analysis by imposing restrictions on the matrices \mathbf{A} and \mathbf{B} in the model form

$$\mathbf{A}y_t = A_1^*y_{t-1} + \dots + A_p^*y_{t-p} + B_0^*x_t + \dots + B_q^*x_{t-q} + C^*D_t + \mathbf{B}\varepsilon_t \quad (8)$$

Here the structural errors ε_t are assumed to be white noise with $(0, I_K)$. The coefficient matrices are structural coefficients which may be different from the reduced form coefficients in (1). The point of departure for a structural analysis is a reduced form model, however. Therefore a reduced form model has to be specified before the *SVAR* analysis can be entered. In the SVAR analysis only restrictions for \mathbf{A} and \mathbf{B} can be added. The reduced form residual u_t is recovered from the structural model as $u_t = \mathbf{A}^{-1}\mathbf{B}\varepsilon_t$ so that $\Sigma_u = \mathbf{A}^{-1}\mathbf{B}\mathbf{B}'\mathbf{A}^{-1'}$.

JMulTi offers three versions of the AB model, an A model where $\mathbf{B} = I_K$, a B model where $\mathbf{A} = I_K$, and a general AB model where restrictions can be placed on both matrices. In addition a Blanchard-Quah model with restrictions placed on the long-run effects of shocks is available (see below).

For the A model and for the B model at least $K(K-1)/2$ restrictions have to be imposed for identification of a system with K endogenous variables. For the AB model, at least $K^2 + K(K-1)/2$ restrictions are needed (see Breitung, Brüggemann and Lütkepohl (2004) for examples).

Estimation is done by maximum likelihood using a scoring algorithm (see Amisano and Giannini (1997) or Breitung et al. (2004)). If the algorithm does not converge, some manual fine tuning may be necessary. If an overidentified model is estimated, the value of a likelihood ratio statistic

$$LR = T \left(\log \det(\tilde{\Sigma}_u^r) - \log \det(\tilde{\Sigma}_u) \right)$$

is also reported. Here $\tilde{\Sigma}_u$ is the ML estimator of the reduced form model and $\tilde{\Sigma}_u^r$ is the corresponding estimator obtained from the restricted structural form estimation.

In the Blanchard-Quah model $\mathbf{A} = I_K$ and the matrix of long-run effects

$$(I_K - A_1 - \dots - A_p)^{-1}\mathbf{B}$$

is assumed to be lower-triangular. In other words, the second residual cannot have a long-run impact (has a zero long-run impact) on the first variable, the third residual cannot have a long-run impact on the first and second variable, etc.. To ensure that plausible restrictions are obtained it may be necessary to adjust the order of the variables. This has to be done in the *Specification* panel. Estimation of the Blanchard-Quah model is done by a Choleski decomposition of the matrix

$$(I_K - \hat{A}_1 - \dots - \hat{A}_p)^{-1}\hat{\Sigma}_u(I_K - \hat{A}_1' - \dots - \hat{A}_p')^{-1},$$

where a hat indicates a reduced form estimate.

Once an SVAR model has been estimated, *SVAR IRA* (impulse response analysis) and *SVAR FEVD* (forecast error variance decomposition) are activated and can be used as described in Sec. 10, 11 and 12.

13.1 Implementation in JMulTi

To access the SVAR analysis, estimate a reduced form VAR model first and then go to *SVAR*→*SVAR Estimation*. Then choose between the Blanchard-Quah and AB models. If the Blanchard-Quah model is chosen, restrictions are imposed automatically and the model can be estimated directly. If standard errors for the estimated long-run coefficients are desired, they can be computed by a bootstrap. Choose the number of bootstrap replications and select the appropriate button to display the estimates.

If an AB model is chosen, specify the kind of model you wish to work with (A, B or AB) and impose the restrictions by clicking on the relevant elements of the matrices activated in the panel. You can also edit coefficients manually after checking the appropriate box. The model will be estimated upon pressing the *Execute* button. If there are convergence problems, press the *Optimization Settings* button and adjust the settings in the panel. Remember, however, that convergence problems can also be due to inappropriate restrictions.

As mentioned earlier, once the SVAR model has been estimated successfully, *SVAR IRA* and *SVAR FEVD* are activated. They can be accessed by going to *SVAR*→*SVAR IRA* or *SVAR*→*SVAR FEVD*.

14 Forecasting with VAR Processes

14.1 Background

Forecasts are based on conditional expectations assuming independent white noise u_t . In other words, an h -step forecast at time T is

$$y_{T+h|T} = A_1 y_{T+h-1|T} + \cdots + A_p y_{T+h-p|T} + B_0 x_{T+h} + \cdots + B_q x_{T+h-q} + CD_{T+h}. \quad (9)$$

The forecasts are computed recursively for $h = 1, 2, \dots$, starting with

$$y_{T+1|T} = A_1 y_T + \cdots + A_p y_{T+1-p} + B_0 x_{T+1} + \cdots + B_q x_{T+1-q} + CD_{T+1}$$

for $h = 1$. Notice that values for the exogenous variables have to be supplied for the forecast period.

The corresponding forecast errors are

$$y_{T+h} - y_{T+h|T} = u_{T+h} + \Phi_1 u_{T+h-1} + \cdots + \Phi_{h-1} u_{T+1},$$

where

$$\Phi_s = \sum_{j=1}^s \Phi_{s-j} A_j, \quad s = 1, 2, \dots,$$

with $\Phi_0 = I_K$ and $A_j = 0$ for $j > p$ (see Lütkepohl (1991)). Thus, the forecast errors have zero mean and, hence, the forecasts are unbiased. Moreover the joint forecast error covariance matrix for all forecasts up to horizon h is

$$\text{Cov} \left(\begin{bmatrix} y_{T+1} - y_{T+1|T} \\ \vdots \\ y_{T+h} - y_{T+h|T} \end{bmatrix} \right) = \begin{bmatrix} I & 0 & \cdots & 0 \\ \Phi_1 & I & & 0 \\ \vdots & & \ddots & \vdots \\ \Phi_{h-1} & \Phi_{h-2} & \cdots & I \end{bmatrix} (\Sigma_u \otimes I_h) \begin{bmatrix} I & 0 & \cdots & 0 \\ \Phi_1 & I & & 0 \\ \vdots & & \ddots & \vdots \\ \Phi_{h-1} & \Phi_{h-2} & \cdots & I \end{bmatrix}' \quad (10)$$

Assuming normally distributed disturbances, these results can be used for setting up forecast intervals for any linear combination of these forecasts. In particular, if any of the variables is in first differenced form, a forecast of the undifferenced variable can be obtained by integration. E.g., if $y_t = \Delta z_t = z_t - z_{t-1}$, $z_{T+h|T} = y_{T+h|T} + \cdots + y_{T+1|T} + z_T$.

For the y_t , forecast intervals may be setup as

$$[y_{k,T+h|T} - c_{1-\gamma/2} \sigma_k(h), y_{k,T+h|T} + c_{1-\gamma/2} \sigma_k(h)],$$

where $c_{1-\gamma/2}$ is the $(1 - \frac{\gamma}{2})100$ percentage point of the standard normal distribution, $y_{k,T+h|T}$ denotes the k th component of $y_{T+h|T}$ and $\sigma_k(h)$ is the standard deviation of the h -step forecast error for the k th component of y_t . Forecast intervals of linear combinations of y_{T+1}, \dots, y_{T+h} may be set up analogously, using (10) for determining the relevant standard deviation.

In practice, the forecasts are based on estimated parameters. This feature may be taken into account by correcting the forecast error variances accordingly, as in Lütkepohl (1991), Sec. 3.5. Notice, however, that the underlying asymptotic theory assumes a stable, stationary process.

14.2 Forecasting in JMulTi

After a model was estimated, the forecast panel is accessible by selecting *Forecasting*→*Forecast Variables* from the menu. It is possible to generate level forecasts as well as level forecasts given that the underlying series is in first differences.

Horizon determines for how many periods forecasts should be computed

CI coverage (%) allows you to choose the coverage level of the forecast intervals

Variables to forecast select the variables that are to be forecasted from the list

Deterministics Because deterministic variables need to be available for each period $T + 1, \dots, T + h$, this table presents the available values that are extrapolated from the actual deterministic series. Extrapolation works well for trend, intercept and seasonal dummies. Other deterministic variables are set to 0, shift dummies are recognized and set to the value they had in T . It is possible to edit the values used for the forecast. If the values of the deterministic series are available for some or all of the h forecast periods, these values are automatically used.

Exogenous Because exogenous variables need to be available for each forecast period $T + 1, \dots, T + h$, this table presents the values used. If values of the exogenous series are available for some or all of the h forecast periods, these values are automatically used. If there are no values available they are automatically set to 0 and have to be edited manually by the user.

Asymptotic CI If selected, the exact asymptotic forecast confidence intervals are estimated, see Lütkepohl (1991), Sec. 5.2.6. and Sec. 10.5

Start date of plot (level) adjusts the date from which the original series should be graphed including the computed forecast. This date does not affect undifferenced forecasts which are only graphed from time T

Forecast generates a forecast resulting in a graph as well as text output on the *Text* tab

Undifferenced Forecast generates a level forecast assuming that the underlying series are in first differences. Exact asymptotic CIs are not available if this option is chosen

Configure Undifferenced Forecast to generate an undifferenced forecast, the values of the endogenous variables in levels for time t have to be given. This panel presents a selection mechanism that allows either to specify the original level series or to manually edit the values needed

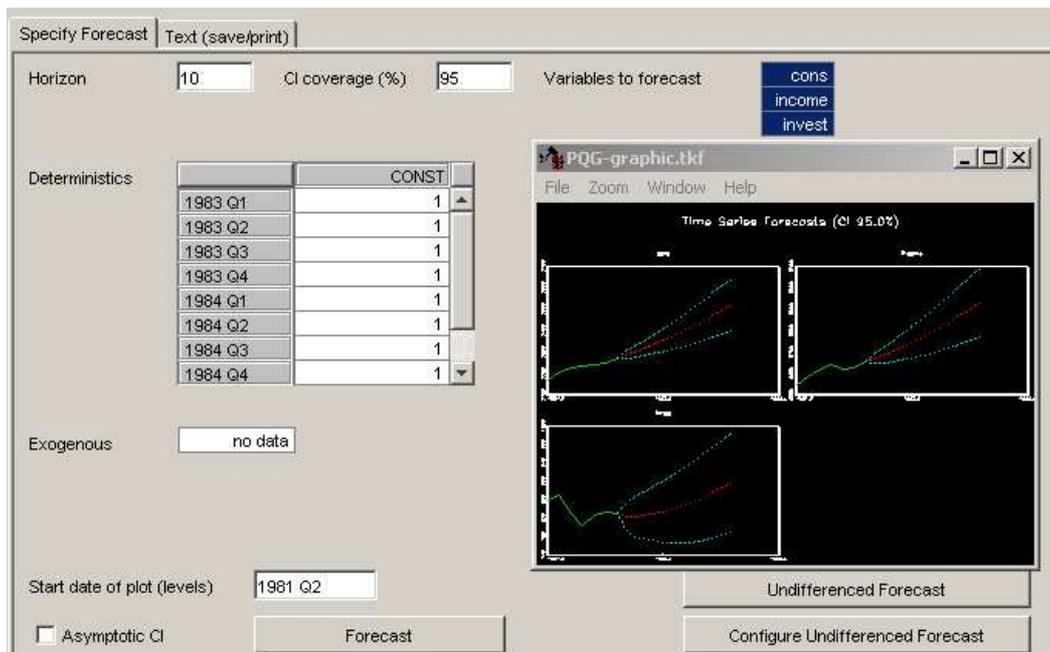


Figure 18: Forecasting in JMulti

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